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Social distancing in the classroom

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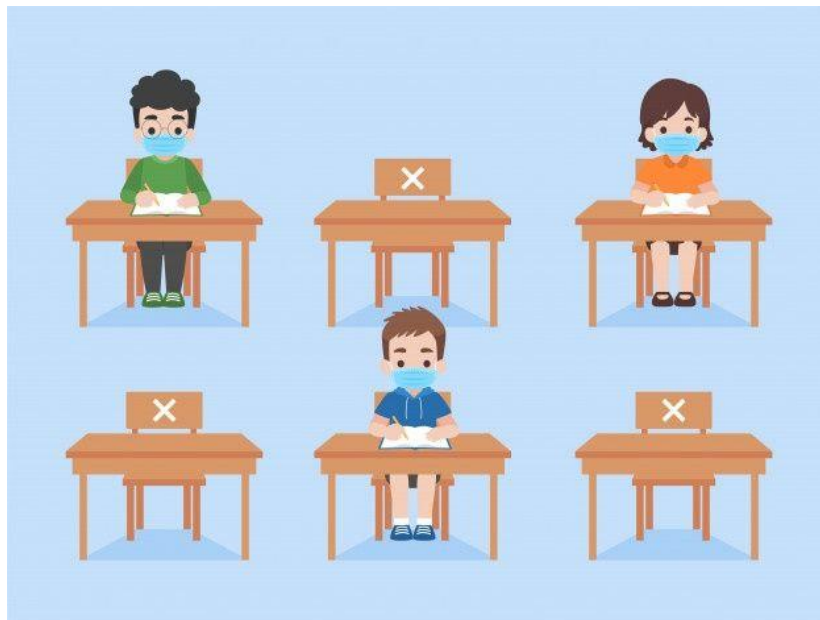
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Abstract

Our research deals with arranging a certain number of students and a teacher in a classroom, while maintaining the social distance between the people in the room. Having the dimensions of the class and the length of the distance that must be kept between the students, we have to find an optimal method of arrangement, so that we can introduce as many people in the class as possible.



The problem

If a classroom is of a rectangular shape $8\text{ m} \times 14\text{ m}$, what would be the maximum number of students, plus a teacher, who could fit in this room while keeping a social distance of 1.5 m ?

How does the number of students change if the social distancing becomes 2 m ?

Generalize to a classroom with a rectangular size $l \times L$ and a social distance d between students.

For a class of 30 students, what would be the minimum requirements for a classroom that would accommodate everyone and maintain social distance?

You can also consider other classroom shapes, such as trapezoidal, with the larger base in front of the class.

For a start, we wanted an optimal placement method, which results in as many people as possible that could fit in the classroom, respecting the social distance.

Therefore, to solve the problem, it was necessary to cover the surface of the class with different geometric shapes (triangles, squares, hexagons, circles), in order to discover the most favorable variant, the one that offers us the largest number of people.

We made the schematic representations of the rectangular surface, using an application called GeoGebra and represented each person in the class with a colored dot. Having the drawings, we compared the number of people who resulted from each drawing and decided which of the methods gives us the best coverage.

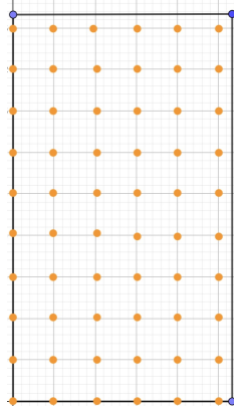
Furthermore, we considered that the students are placed in one-person desks, these having negligible dimensions. We also neglected the distances between people and the walls of the room.

Solution of the problem

If a classroom is of a rectangular shape $8\text{ m} \times 14\text{ m}$, what would be the maximum number of students, plus a teacher, who could fit in this room while keeping a social distance of 1.5 m ?

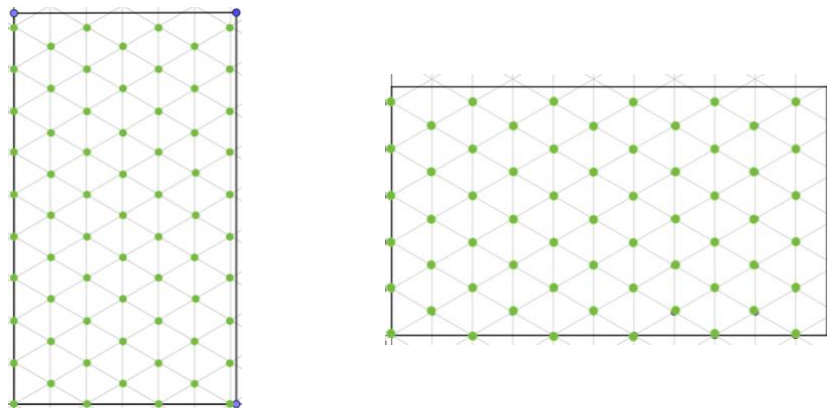
We considered four methods of placing people in the classroom. We divided the classroom area into squares, triangles, hexagons, and circles, in order to find out which one results the biggest number of people.

Option 1: We fill the rectangular surface with 1.5m side squares, each vertex of the square representing the position of a person in the class. Using the GeoGebra application, we got the following distribution:



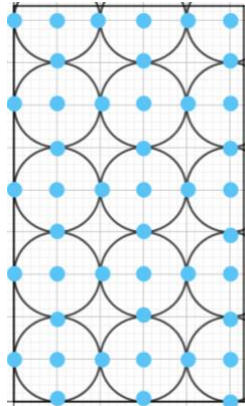
Hence, we get the possibility of placing *60 people (59 students and 1 teacher)*.

Option 2: On the surface of the class, we place equilateral triangles with a side of 1.5m , each vertex representing the position of a person.



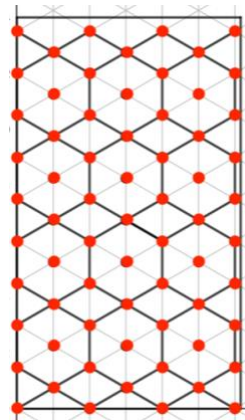
Using the first representation, we get a number of *67 people (66 students and 1 teacher)* that could fit in the room, while using the second, we get a total number of *61 people (60 students and 1 teacher)*. So, the most convenient representation is the first, obtaining *67 people (66 students and 1 teacher)*.

Option 3: Using circles with a radius of 1.5m, we fill the surface of the class.



By this method, we can arrange *44 people (43 students and 1 teacher)* in the classroom.

Option 4: We cover the rectangular surface of the classroom with regular hexagons of side 1.5m.



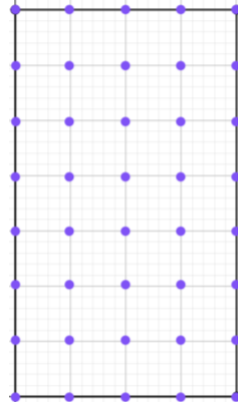
In this case, we can place *67 people (66 students and 1 teacher)* in the room.

In conclusion, the most favorable methods of placing people in the classroom are the ones with equilateral triangles (option 2, representation 1) or with the regular hexagons (option 4), as they result the same number of people. As mentioned above, we get a number of ***67 people (66 students and 1 teacher)*** who could be placed in the classroom.

How does the number of students change if the social distancing becomes 2 m?

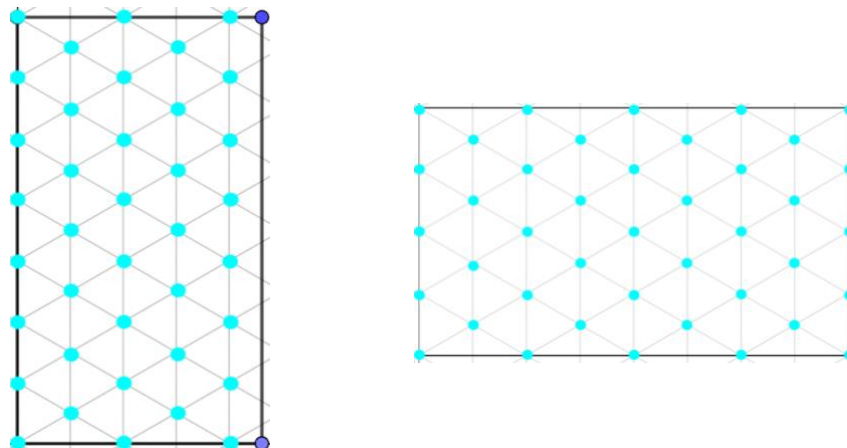
As in the case of the distance of 1.5 m, we have 4 possible options for arranging the people in the classroom.

Option 1: we use the method with squares, this time having the side of the square equal to 2m, in order to keep the distance.



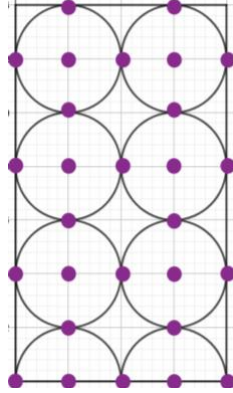
In this way, we can place *40 people (39 students and 1 teacher)* in the classroom.

Option 2: On the surface of the class, we place equilateral triangles with a side of 2m, each vertex representing the position of a person.



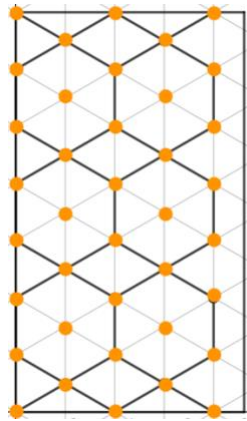
Using the first representation, we get a number of *38 people (37 students and 1 teacher)* that could fit in the room, while using the second, we get a total number of *41 people (40 students and 1 teacher)*. So, the most convenient representation is the second, obtaining 41 people (40 students and 1 teacher).

Option 3: We fill the surface with circles with a radius of 2m.



By this method, we can arrange only 28 people (27 students and 1 teacher).

Option 4: We cover the rectangular surface of the classroom with regular hexagons of side 2m.



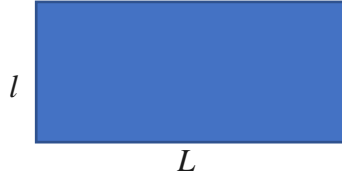
In this case, the total number of people is 38 (37 students and 1 teacher).

Following the analysis of the variants discussed above, the optimal variant is the one in which the coverage is made with equilateral triangles (option 2, representation 2), thus generating a number of **41 people (40 students and 1 teacher)** that could be arranged in the classroom, while maintaining the social distance of 2m.

Generalize to a classroom with a rectangular size $l \times L$ and a social distance d between students.

As we see from the above examples, the coverage of the surface of the classroom with circles is not the best option, as it does not allow for the maximum seat options for the students. Therefore, we are going to use the equilateral triangles option.

Let us assume $l, L > 0$ and $l < L$.



Case 1. We discretize the length L , and then cover the classroom with a triangular grid.

(A) The first thing we do is to consider $d = 1.5m$ and equilateral triangles coverage with a side equal of $1.5m$. Such a triangle has the height $h_{\Delta} = \frac{1.5\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$.

The number of distances on one length of the rectangular is $\left[\frac{L}{1.5} \right]$, where by $[x]$ we have denoted the integer part of x (the floor function). On the odd numbered columns we are going to place $\left[\frac{L}{1.5} \right] + 1$ people, and on the even numbered columns we will place $\left[\frac{L}{1.5} \right]$ people. We denote by $k_1 = \left[\frac{l}{\frac{3\sqrt{3}}{4}} \right] + 1$ the number of columns, where $\left[\frac{l}{\frac{3\sqrt{3}}{4}} \right]$ represents the number of distances on one width of the rectangular surface, and by n_1 the maximum number of people which can be seated in the rectangular classroom with size $l \times L$.

We also notice that, for two distances on the width, we have

$$\left[\frac{L}{1.5} \right] + 1 + \left[\frac{L}{1.5} \right] = 2 \cdot \left[\frac{L}{1.5} \right] + 1.$$

Thus, if k_1 is even, then

$$n_1 = \frac{k_1}{2} \left(2 \cdot \left[\frac{L}{1.5} \right] + 1 \right) = k_1 \cdot \left[\frac{L}{1.5} \right] + \frac{k_1}{2}.$$

If k_1 is odd, then

$$n_1 = \frac{k_1 - 1}{2} \cdot \left(2 \cdot \left[\frac{L}{1.5} \right] + 1 \right) + \left[\frac{L}{1.5} \right] + 1.$$

As we know, for $(\forall) x \in N$, $\left[\frac{x+1}{2} \right] = \frac{x}{2}$, if x is even, and $\left[\frac{x+1}{2} \right] = \frac{x+1}{2}$, if x is odd. Using this observation we obtain that the maximum number of people is

$$n_1 = k_1 \cdot \left[\frac{L}{1.5} \right] + \left[\frac{k_1 + 1}{2} \right],$$

with $k_1 = \left\lceil \frac{l}{\frac{3\sqrt{3}}{4}} \right\rceil + 1$.

Let us particularize the above formulas for $l = 8$ and $L = 14$. Because $k_1 = \left\lceil \frac{8}{\frac{3\sqrt{3}}{4}} \right\rceil + 1 = 7$ is odd, and $\left\lceil \frac{14}{1.5} \right\rceil = 9$, then $n_1 = 7 \cdot 9 + 4 = 67$ people, as we have observed in the previous graphical example.

(B) If we consider $d = 2m$ and equilateral triangles coverage with a side equal to $2m$ with the height $h_\Delta = \frac{2\sqrt{3}}{2} = \sqrt{3}$, the number of distances on one length of the rectangular is $\left\lceil \frac{L}{2} \right\rceil$ and $k_1 = \left\lceil \frac{l}{\sqrt{3}} \right\rceil + 1$. Using the formula from above, for $l = 8$ and $L = 14$, we get

$$\left\lceil \frac{14}{2} \right\rceil = 7, k_1 = \left\lceil \frac{8}{\sqrt{3}} \right\rceil + 1 = 5 \text{ and } n_1 = 5 \cdot 7 + 3 = 38 \text{ people.}$$

(C) We can now generalize the formula obtained for the particular cases $d = 1.5m$ and $d = 2m$. If we consider that the minimum distance between two people is d and we will use equilateral triangles with the sidelength equal to the same value, d , then number of people seated on the odd columns is $\left\lceil \frac{L}{d} \right\rceil + 1$ and on the even ones is $\left\lceil \frac{L}{d} \right\rceil$. The height of a such rectangle is $h_\Delta = \frac{d\sqrt{3}}{2}$, and $k_1 = \left\lceil \frac{l}{\frac{d\sqrt{3}}{2}} \right\rceil + 1 = \left\lceil \frac{2l}{d\sqrt{3}} \right\rceil + 1$ is the number of columns.

If k_1 is even, the number of people is

$$n_1 = \frac{k_1}{2} \left(2 \cdot \left\lceil \frac{L}{d} \right\rceil + 1 \right) = k_1 \cdot \left\lceil \frac{L}{d} \right\rceil + \frac{k_1}{2}.$$

If k_1 is odd, we have

$$n_1 = \frac{k_1 - 1}{2} \cdot \left(2 \cdot \left\lceil \frac{L}{d} \right\rceil + 1 \right) + \left\lceil \frac{L}{d} \right\rceil + 1,$$

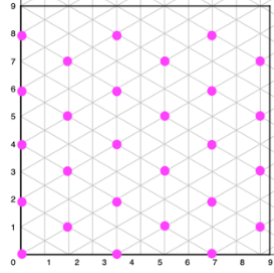
with $k_1 = \left\lceil \frac{2l}{d\sqrt{3}} \right\rceil + 1$. Because $\left\lceil \frac{k_1+1}{2} \right\rceil$ is $\frac{k_1}{2}$ or $\frac{k_1+1}{2}$, depending on the parity of k_1 , the previous two formulae can be rewritten into a single one

$$n_1 = k_1 \cdot \left\lceil \frac{L}{d} \right\rceil + \left\lceil \frac{k_1 + 1}{2} \right\rceil. \quad (1)$$

Let us test the above formula for some particular cases:

1. For $l = 8, L = 14$ and $d = 1.5$, we evaluate $k_1 = \left\lceil \frac{2l}{d\sqrt{3}} \right\rceil + 1 = \left\lceil \frac{16}{1.5\sqrt{3}} \right\rceil + 1 = 7$, which is odd, and $\left\lceil \frac{L}{d} \right\rceil = \left\lceil \frac{14}{1.5} \right\rceil = 9$. We obtain $n_1 = 7 \cdot 9 + 4 = 63 + 4 = 67$ people.

- Let us consider now $l = 8, L = 14$ and $d = 2$. Then, $k_1 = \left\lfloor \frac{2l}{d\sqrt{3}} \right\rfloor + 1 = \left\lfloor \frac{16}{2\sqrt{3}} \right\rfloor + 1 = 5$, which is odd, and $\left\lfloor \frac{L}{d} \right\rfloor = \left\lfloor \frac{14}{2} \right\rfloor = 7$. Thus, $n_1 = 5 \cdot 7 + 3 = 35 + 3 = 38$ people.
- For $l = 9, L = 9$ and $d = 2$, we get $k_1 = \left\lfloor \frac{2l}{d\sqrt{3}} \right\rfloor + 1 = \left\lfloor \frac{18}{2\sqrt{3}} \right\rfloor + 1 = 6$, which is even and $\left\lfloor \frac{L}{d} \right\rfloor = \left\lfloor \frac{9}{2} \right\rfloor = 4$. We obtain $n_1 = 6 \cdot 4 + 3 = 24 + 3 = 27$ people, as we can also observe in the following figure:



Case 2. We discretize the width l , and then cover the classroom with a triangular grid.

The number of distances on one width (column) of the rectangular shape is $\left\lfloor \frac{l}{d} \right\rfloor$. On the odd numbered columns will be $\left\lfloor \frac{l}{d} \right\rfloor + 1$ people and on the even numbered columns we will use $\left\lfloor \frac{l}{d} \right\rfloor$ people. $\left\lfloor \frac{L}{d\sqrt{3}} \right\rfloor$ represents the number of distances on one length of the rectangular surface. With $k_2 = \left\lfloor \frac{2L}{d\sqrt{3}} \right\rfloor + 1$, the number of column, we get n_2 , the maximum number of people,

$$n_2 = k_2 \cdot \left\lfloor \frac{l}{d} \right\rfloor + \left\lfloor \frac{k_2 + 1}{2} \right\rfloor. \quad (2)$$

We consider some particular cases:

- For $l = 8, L = 14$ and $d = 1.5$, we evaluate $k_2 = \left\lfloor \frac{2L}{d\sqrt{3}} \right\rfloor + 1 = \left\lfloor \frac{28}{1.5\sqrt{3}} \right\rfloor + 1 = 11$, which is odd, and $\left\lfloor \frac{l}{d} \right\rfloor = \left\lfloor \frac{8}{1.5} \right\rfloor = 5$. We obtain $n_2 = 11 \cdot 5 + 6 = 55 + 6 = 61$ people.
- Let us consider now $l = 8, L = 14$ and $d = 2$. Then, $k_2 = \left\lfloor \frac{2L}{d\sqrt{3}} \right\rfloor + 1 = \left\lfloor \frac{28}{2\sqrt{3}} \right\rfloor + 1 = 9$, which is odd, and $\left\lfloor \frac{l}{d} \right\rfloor = \left\lfloor \frac{8}{2} \right\rfloor = 4$. Thus, $n_2 = 9 \cdot 4 + 5 = 36 + 5 = 41$ people.

Taking into consideration the above results, we suggest that the optimal arrangement of the students and their teacher is obtained using equilateral triangles, in one of the two cases that we have studied.

Conclusion: Based on the previous calculations, the optimal number of the people sitting in a $L \times l$ rectangular classroom is the maximum value of the values obtained in (1) and (2), that is:

$$n = \max\{n_1, n_2\}$$

$$= \max\left\{k_1 \cdot \left[\frac{L}{d}\right] + \left[\frac{k_1+1}{2}\right], k_2 \cdot \left[\frac{L}{d}\right] + \left[\frac{k_2+1}{2}\right]\right\},$$

where

$$k_1 = \left[\frac{2l}{d\sqrt{3}}\right] + 1 \quad \text{and} \quad k_2 = \left[\frac{2L}{d\sqrt{3}}\right] + 1.$$

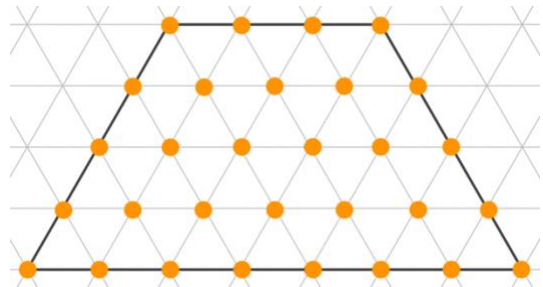
Here, $[x]$ denotes the integer part of x (the floor function).

For a class of 30 students, what would be the minimum requirements for a classroom that would accommodate everyone and maintain social distance?

Considering the placement of 30 students, the representation shown below is the most efficient way to arrange them in the classroom, while keeping the social distance. For this arrangement, we used the equilateral triangles method, and we made the representation of the surface in GeoGebra app. We observe that the surface has the form of an isosceles trapezoid.

We denote by:

- d – the length of the social distance
- B – the larger base of the trapezoid
- b – the smaller base of the trapezoid
- H – the height of the trapezoid
- h_{Δ} – the height of an equilateral triangle



$$B = 7 \cdot d \, m$$

$$b = 3 \cdot d \, m$$

$$h_{\Delta} = \frac{d\sqrt{3}}{2} \, m$$

$$H = 4 \cdot h_{\Delta} = 4 \cdot \frac{d\sqrt{3}}{2} \, m = 2d\sqrt{3} \, m$$

$$\mathcal{A} = \frac{(b+B) \cdot H}{2} = \frac{(7 \cdot d + 3 \cdot d) \cdot 2d\sqrt{3}}{2} = \frac{10 \, d \cdot 2d\sqrt{3}}{2} = 10d^2\sqrt{3} \, m^2$$

Let us test the above formula for some particular cases:

1. If we consider $d = 1.5m$, we get

$$\mathcal{A} = 10d^2\sqrt{3} m^2 = 10 \cdot 1.5^2 \cdot \sqrt{3} = 10 \cdot 2.25 \cdot \sqrt{3} \cong 39 m^2$$

2. For $d = 2m$, we obtain

$$\mathcal{A} = 10d^2\sqrt{3} m^2 = 10 \cdot 2^2 \cdot \sqrt{3} = 10 \cdot 4 \cdot \sqrt{3} \cong 69 m^2$$

So, the minimum requirements for a classroom that would accommodate everyone and maintain social distance d would be to have the area equal to $10d^2\sqrt{3} m^2$.

Conclusions

At first, we thought it would be easy to arrange the students in a classroom, while keeping some distance between them. When we actually started solving the problem, we realized that it wasn't as easy as it seemed when we first read it. However, what motivated us was that it was describing a situation we have been facing since March 2020. We realized that our teachers at school had to do the same arrangement in our classroom, in order to stay safe when we were going to school. After we have finished solving the problem, we have seen that it was just about arranging people in the shapes of geometric figures and, finally, we obtained the desired arrangement, which included the maximum possible number of people.