

ENI GAME AND PROBABILITY

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INTRODUCTION

Throughout the school year 2019/2020 we were taking part in a project Erasmus+ called "Maths & Languages". Our group worked on the subject called „ENI game”.

RULES OF THE GAME:

- There are two players: H and S.
- The board consists of n tiles. The player H chooses a tile under which he hides his treasure. He also has 1 black stone.
- Then player S tries to find the treasure. He indicates his first guess with a white stone.
- The player H puts the black stone on the board on a tile that is **Empty** (without treasure) and **Non-Indicated** (by the white stone). Now the player S can either change his first guess or not. If he decides to change it, he moves his white stone into the place of his "second choice" - it cannot be the tile with the black stone.
- If the white stone is located on the tile with the treasure, the player S wins. Otherwise the player H will be the winner.

H – the person that hides the treasure
S – the person that searches for the treasure
 n ($3 \leq n \leq 10$) – number of tiles

PROBLEM:

Which choice is better for H: keeping the first decision or not?

OUR GAMES:

Our team played the game many times to see which option would be more effective - changing the position of the white stones or not changing it. We checked the results with or without changing in a different number of tiles. We ended up with a conclusion that changing the position increases the winning rate and the absence of changing increases a losing rate.

EXCEL SIMULATION:

This is the simulation of ENI game, that we made in Excel. Program itself plays next games and shows us results like amount of wins with/without changing first choice and amount of losses with/without changing first choice. Moreover Excel shows us these results as percentage winning ratio in both cases (with/without change).

F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
Treasure	First choice	p2	p3	p4	p5			First covered tile	p1	p2	p3	p4	p5	Final choice	change?	win?
1	1	2	3	4	5	0	1	5	1	2	3	4	0	1	FALSZ	PRAWDA
1	5	2	3	4	0	1	3	2	1	0	3	4	5	3	PRAWDA	FALSZ
1	4	2	3	0	5	1	1	5	1	2	3	4	0	1	PRAWDA	PRAWDA
1	2	0	3	4	5	1	1	5	1	2	3	4	0	1	PRAWDA	PRAWDA
1	1	2	3	4	5	0	2	4	1	2	3	0	5	3	PRAWDA	FALSZ
1	2	0	3	4	5	1	2	4	1	2	3	0	5	1	PRAWDA	PRAWDA

I'm going to try to explain how it works, basing on example of situation when N=5.

- Column F picks tile with treasure, in our simulation it's always tile no. 1.
- Column G takes random number from 1 to 5, to pick first choice.
- Columns H-K check which tiles aren't first choice (those are marked as 0) and which tiles can be covered (those are numbers from 2 to 5). We don't check the first tile because it's always tile with treasure, so it can't be covered.
- Column L checks how many of columns H-K equal "0" and depending on that if it's zero or one column with "0" the column M randomly picks number from 1-3 (in case of one "0") or from 1-4 (in case of no "0").
- Then column N basing on two previous ones, picks the covered tile.
- Columns O-S do the same thing as columns H-K so once again they check which tiles are available for optional change of choice.
- Column T picks maximum value in turns of random number from 1 to 4. That's final choice.

Comparing values from columns G and T Excel knows if in this case there was change of choice and checking if final choice equals 1, it knows if the game was won-

Here are final results of simulation for N=3, 4, 5.

N=3			
with change		without change	
win	loss	win	loss
70,04%	29,96%	36,85%	63,15%

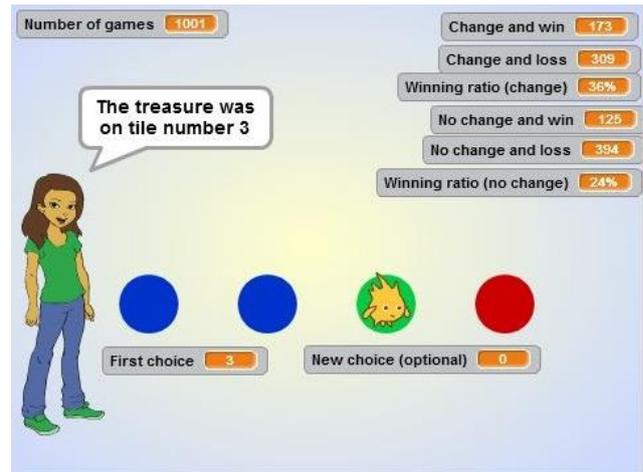
N=4			
with change		without change	
win	loss	win	loss
39,49%	60,51%	25,68%	74,32%

N=5			
with change		without change	
win	loss	win	loss
29,17%	70,83%	16,59%	83,41%

SCRATCH PROGRAMME

Additionally, we have recreated the ENI game in the Scratch program, in order to see if the results were different from our Excel games. We have ran a vast amount of games with different value of n (3,4,5) to compare.

The results showed, that the number of won games was almost equal to the number of won Excel games, which maintained the correctness of Excel results and conclusions we have drawn from them.



Scratch simulation gave us following results: for N=3 winning ratio 63% with change and 32 % without change, for N=4 winning ratio 36% with change and 24% without change, for N=5 winning ratio 30% with change and 18% without change.

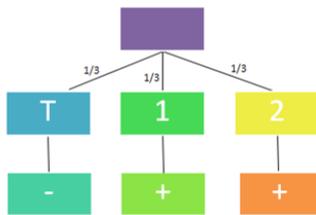
Comparing results from Scratch to results from our Excel simulation showed, that they're very similar to each other, making our simulations valid.

PROBABILITY RESEARCH:

To find a formula we tried using the tree diagram method. It allowed us to calculate the probability of winning when we change our decision with a given n.

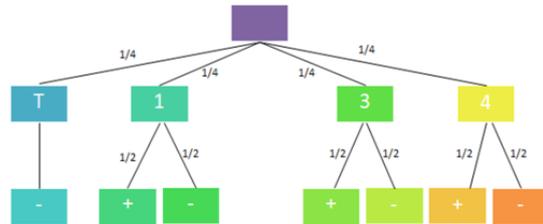
With the help of our teachers we finally managed to draw the diagrams. Here is an example:

n=3



$$P(+)=\frac{1}{3}\cdot 1+\frac{1}{3}\cdot 1=\frac{2}{3}$$

n=4

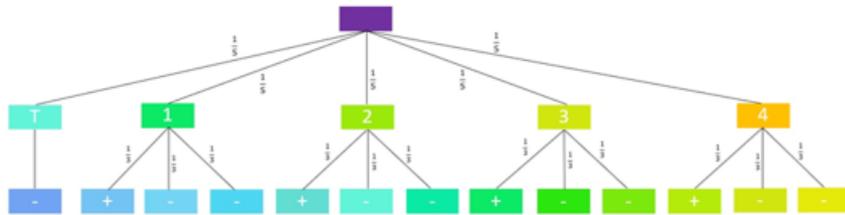


$$P(+)=\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{4}\cdot\frac{1}{2}=\frac{3}{8}$$

$$P(+)=\frac{1}{4}\cdot\frac{1}{2}\cdot 3=\frac{3}{8}$$

After drawing a diagram we were able to calculate the probability of winning the game. For the parameters n=3 it was 2/3. (the diagram on the left). For n=4, the probability of winning equalled 3/8 (the diagram on the right).

For n=5, the probability of winning equalled 4/15 (the diagram below).



$$P(+)=\frac{1}{5}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{3}=\frac{4}{15}$$

$$P(+)=\frac{1}{5}\cdot\frac{1}{3}\cdot 4=\frac{4}{15}$$

Through analysis of the numbers we managed to come up with a formula that matched all of our calculations.

$$(n-1)\cdot\frac{1}{n}\cdot\frac{1}{n-2}$$

COMPARISON

n	Excel simulation	Scratch	formula
3	69%	63%	67%
4	38%	36%	38%
5	28%	30%	27%
6	21%	–	21%
7	18%	–	18%
8	15%	–	13%
9	14%	–	13%

WHAT IF WE USE MORE BLACK STONES?

$$(n - 1) \cdot \frac{1}{n} \cdot \frac{1}{n - (k + 1)}$$

Here is the comparison of our Excel results and calculations based on the formula for k=2:

n	Excel simulation	formula
4	71,88%	75,00%
5	52,63%	40,00%
6	30,00%	27,78%
7	33,33%	21,43%
8	10,00%	17,50%
9	16,67%	14,81%

<p>n - number of tiles</p> <p>k - number of black stones</p>
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OVERVIEW:

We played the game a couple of times and noted the results. Counted the probability of winning with both strategies based on our results. Compared the figures to the mathematically calculated probabilities and finally come up with a formula. We came to the conclusion that the probability of winning is higher when you change your first choice.