# Social distancing in the classroom

Year 2020 – 2021

**Students**: Moşneagu Magda (student, 7th grade), Nistor Călin-Ioan (student, 7th grade), Puşcasu Ilinca-Maria (student, 7th grade), Săveanu Andreea-Florina (student, 7th grade)

**School:** "Costache Negruzzi" National College of Iași, Romania

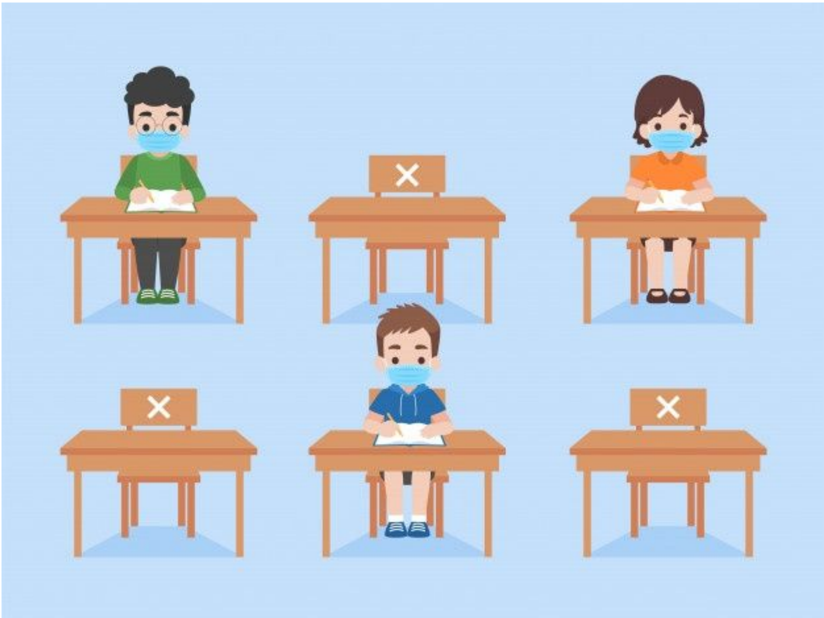
**Teacher:** Ph.D. Ioana Cătălina Anton, "Costache Negruzzi" National College of Iași, Romania

**Researcher**: Ph.D. Iulian Stoleriu, University "Alexandru Ioan Cuza" of Iasi, Romania

**Keywords**: triangles, squares, area, social distance

## *Abstract*

Our research deals with arranging a certain number of students and a teacher in a classroom, while maintaining the social distance between the people in the room. Having the dimensions of the class and the length of the distance that must be kept between the students, we have to find an optimal method of arrangement, so that we can introduce as many people in the class as possible.



## *The problem*

If a classroom is of a rectangular shape 8 m × 14 m, what would be the maximum number of students, plus a teacher, who could fit in this room while keeping a social distance of 1.5 m?

How does the number of students change if the social distancing becomes 2 m?

Generalize to a classroom with a rectangular size l × L and a social distance d between students.

For a class of 30 students, what would be the minimum requirements for a classroom that would accommodate everyone and maintain social distance?
You can also consider other classroom shapes, such as trapezoidal, with the larger base in front of the class.

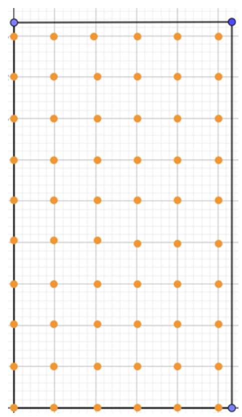
For a start, we wanted an optimal placement method, which results in as many people as possible that could fit in the classroom, respecting the social distance.
Therefore, to solve the problem, it was necessary to cover the surface of the class with different geometric shapes (triangles, squares, hexagons, circles) (1), in order to discover the most favorable variant, the one that offers us the largest number of people.
We made the schematic representations of the rectangular surface, using an application called GeoGebra and represented each person in the class with a colored dot.
Having the drawings, we compared the number of people who resulted from each drawing and decided which of the methods gives us the best coverage (2).
Furthermore, we considered that the students are placed in one-person desks, these having negligible dimensions. We also neglected the distances between people and the walls of the room.

## *Solution of the problem*

***If a classroom is of a rectangular shape 8 m × 14 m, what would be the maximum number of students, plus a teacher, who could fit in this room while keeping a social distance of 1.5 m?***

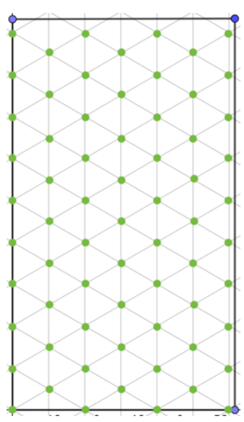
We considered four methods of placing people in the classroom. We divided the classroom area into squares, triangles, hexagons, and circles, in order to find out which one results the biggest number of people.

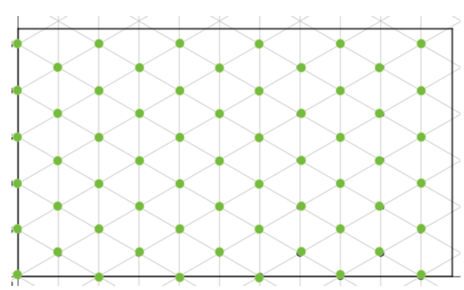
**Option 1**: We fill the rectangular surface with 1.5m side squares (3), each vertex of the square representing the position of a person in the class. Using the GeoGebra application, we got the following distribution:



Hence, we get the possibility of placing *60 people (59 students and 1 teacher)*.

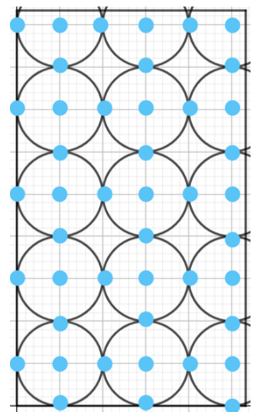
**Option 2**: On the surface of the class, we place equilateral triangles with a side of 1.5m, each vertex representing the position of a person.





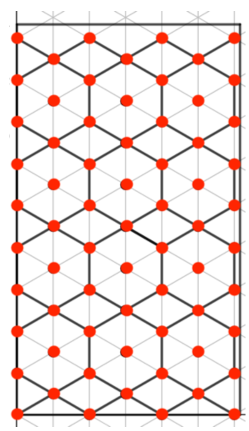
Using the first representation, we get a number of *67 people (66 students and 1 teacher)* that could fit in the room, while using the second, we get a total number of *61 people (60 students* *and 1 teacher)* (4). So, the most convenient representation is the first, obtaining *67 people (66 students and 1 teacher).*

**Option 3**: Using circles with a radius of 1.5m, we fill the surface of the class (5).



By this method, we can arrange *44 people (43 students and 1 teacher*) in the classroom.

**Option 4**: We cover the rectangular surface of the classroom with regular hexagons of side 1.5m (6).
.



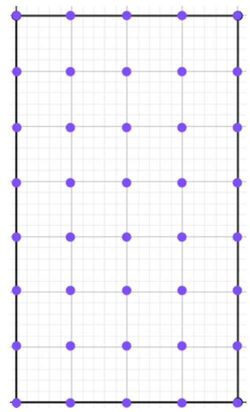
In this case, we can place *67 people (66 students and 1 teacher)* in the room.

In conclusion, the most favorable methods of placing people in the classroom are the ones with equilateral triangles (option 2, representation 1) or with the regular hexagons (option 4), as they result the same number of people. As mentioned above, we get a number of ***67 people (66 students and 1 teacher)*** who could be placed in the classroom (7).

***How does the number of students change if the social distancing becomes 2 m?***

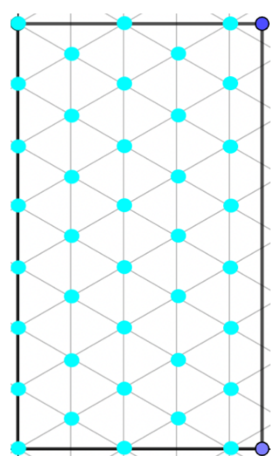
As in the case of the distance of 1.5 m, we have 4 possible options for arranging the people in the classroom (8).

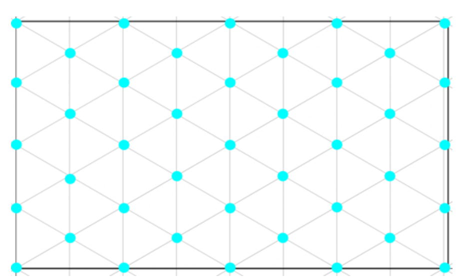
**Option 1**: we use the method with squares, this time having the side of the square equal to 2m, in order to keep the distance.



In this way, we can place *40 people (39 students and 1 teacher)* in the classroom.

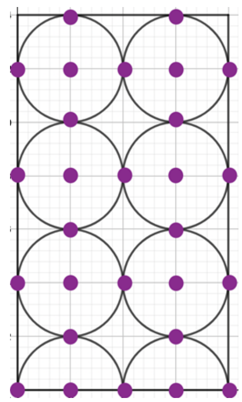
**Option 2**: On the surface of the class, we place equilateral triangles with a side of 2m, each vertex representing the position of a person.





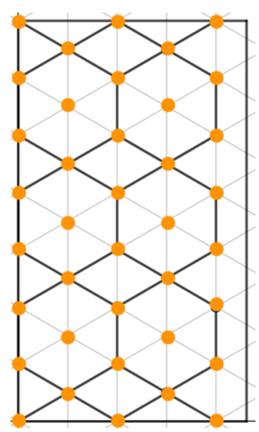
Using the first representation, we get a number of *38 people (37 students and 1 teacher)* that could fit in the room, while using the second, we get a total number of *41 people (40 students* *and 1 teacher)*. So, the most convenient representation is the second, obtaining 41 people (40 students and 1 teacher).

**Option 3**: We fill the surface with circles with a radius of 2m.



By this method, we can arrange only *28 people (27 students and 1 teacher)*.

**Option 4:** We cover the rectangular surface of the classroom with regular hexagons of side 2m.



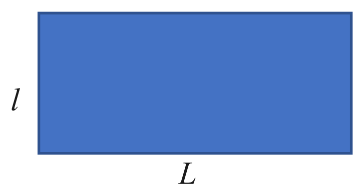
In this case, the total number of people is *38 (37 students and 1 teacher)*.

Following the analysis of the variants discussed above, the optimal variant is the one in which the coverage is made with equilateral triangles (option 2, representation 2), thus generating a number of ***41 people (40 students and 1 teacher)*** that could be arranged in the classroom, while maintaining the social distance of 2m.

***Generalize to a classroom with a rectangular size l × L and a social distance d between students.***

As we see from the above examples, the coverage of the surface of the classroom with circles is not the best option, as it does not allow for the maximum seat options for the students. Therefore, we are going to use the equilateral triangles option (9).

Let us assume $l, L > 0$ and $l < L$ (10).




**Case 1.** We discretize the length $L$, and then cover the classroom with a triangular grid.

(A) The first thing we do is to consider $d = 1.5m$ and equilateral triangles coverage with a side equal of $1.5m$. Such a triangle has the height $h_\Delta = \frac{1.5\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$.

The number of distances on one length of the rectangular is $\left[\frac{L}{1.5}\right]$ (11), where by $[x]$ we have denoted the integer part of $x$ (the floor function). On the odd numbered columns we are going to place $\left[\frac{L}{1.5}\right] + 1$ people, and on the even numbered columns we will place $\left[\frac{L}{1.5}\right]$ persons. We denote by $k_1 = \left[\frac{l}{\frac{3\sqrt{3}}{4}}\right] + 1$ the number of columns, where $\left[\frac{l}{\frac{3\sqrt{3}}{4}}\right]$ represents the number of distances on one width of the rectangular surface, and by $n_1$ the maximum number of people which can be seated in the rectangular classroom with size $l \times L$.

We also notice that, for two distances on the width, we have

$$\left[\frac{L}{1.5}\right] + 1 + \left[\frac{L}{1.5}\right] = 2 \cdot \left[\frac{L}{1.5}\right] + 1.$$

Thus, if $k_1$ is even, then

$$n_1 = \frac{k_1}{2}\left(2 \cdot \left[\frac{L}{1.5}\right] + 1\right) = k_1 \cdot \left[\frac{L}{1.5}\right] + \frac{k_1}{2}.$$

If $k_1$ is odd, then

$$n_1 = \frac{k_1 - 1}{2} \cdot \left(2 \cdot \left[\frac{L}{1.5}\right] + 1\right) + \left[\frac{L}{1.5}\right] + 1.$$

As we know, for (∀) $x \in N, \left[\frac{x+1}{2}\right] = \frac{x}{2}$, if $x$ is even, and $\left[\frac{x+1}{2}\right] = \frac{x+1}{2}$, if $x$ is odd. Using this observation we obtain that the maximum number of people is

$$n_1 = k_1 \cdot \left[\frac{L}{1.5}\right] + \left[\frac{k_1+1}{2}\right],$$

with $k_1 = \left[\frac{l}{\frac{3\sqrt{3}}{4}}\right] +1$.

Let us particularize the above formulas for $l = 8$ and $L = 14$. Because $k_1 = \left[\frac{8}{\frac{3\sqrt{3}}{4}}\right] +1 = 7$ is odd, and $\left[\frac{14}{1.5}\right] = 9$, then $n_1 = 7 \cdot 9 + 4 = 67$ people, as we have observed in the previous graphical example.

(B) If we consider $d = 2m$ and equilateral triangles coverage with a side equal to $2m$ with the height $h_\Delta = \frac{2\sqrt{3}}{2} = \sqrt{3}$ , the number of distances on one length of the rectangular is $\left[\frac{L}{2}\right]$ and $k_1 = \left[\frac{l}{\sqrt{3}}\right] +1$. Using the formula from above, for $l = 8$ and $L = 14$, we get

$$\left[\frac{14}{2}\right] = 7,\ k_1 = \left[\frac{8}{\sqrt{3}}\right] +1 = 5 \text{ and } n_1 = 5 \cdot 7 + 3 = 38 \text{ people } (12).$$

(C) We can now generalize the formula obtained for the particular cases $d = 1.5m$ and $d = 2m$. If we consider that the minimum distance between two people is $d$ and we will use equilateral triangles with the sidelength equal to the same value, $d$, then number of people seated on the odd columns is $\left[\frac{L}{d}\right] +1$ and on the even ones is $\left[\frac{L}{d}\right]$. The height of a such rectangle is $h_\Delta = \frac{d\sqrt{3}}{2}$, and $k_1 = \left[\frac{l}{\frac{d\sqrt{3}}{2}}\right] +1 = \left[\frac{2l}{d\sqrt{3}}\right] +1$ is the number of columns.

If $k_1$ is even, the number of people is

$$n_1 = \frac{k_1}{2}\left(2 \cdot \left[\frac{L}{d}\right] + 1\right) = k_1 \cdot \left[\frac{L}{d}\right] + \frac{k_1}{2}\ .$$

If $k_1$ is odd, we have

$$n_1 = \frac{k_1 - 1}{2} \cdot \left(2 \cdot \left[\frac{L}{d}\right] + 1\right) + \left[\frac{L}{d}\right] + 1,$$

with $k_1 = \left[\frac{2l}{d\sqrt{3}}\right] +1$. Because $\left[\frac{k_1+1}{2}\right]$ is $\frac{k_1}{2}$ or $\frac{k_1+1}{2}$, depending on the parity of $k_1$, the previous two formulae can be rewritten into a single one

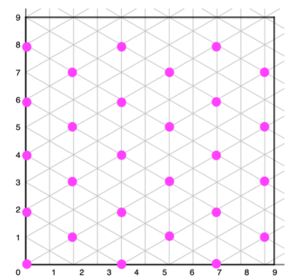
$$n_1 = k_1 \cdot \left[\frac{L}{d}\right] + \left[\frac{k_1+1}{2}\right]. \qquad (1)$$

Let us test the above formula for some particular cases:

1. For $l = 8$, $L = 14$ and $d = 1.5$, we evaluate $k_1 = \left[\frac{2l}{d\sqrt{3}}\right] + 1 = \left[\frac{16}{1.5\sqrt{3}}\right] + 1 = 7$, which is odd, and $\left[\frac{L}{d}\right] = \left[\frac{14}{1{,}5}\right] = 9$. We obtain $n_1 = 7 \cdot 9 + 4 = 63 + 4 = 67$ people.

2. Let us consider now $l = 8$, $L = 14$ and $d = 2$. Then, $k_1 = \left[\frac{2l}{d\sqrt{3}}\right] + 1 = \left[\frac{16}{2\sqrt{3}}\right] + 1 = 5$, which is odd, and $\left[\frac{L}{d}\right] = \left[\frac{14}{2}\right] = 7$. Thus, $n_1 = 5 \cdot 7 + 3 = 35 + 3 = 38$ people.

3. For $l = 9$, $L = 9$ and $d = 2$, we get $k_1 = \left[\frac{2l}{d\sqrt{3}}\right] + 1 = \left[\frac{18}{2\sqrt{3}}\right] + 1 = 6$, which is even and $\left[\frac{L}{d}\right]$ $= \left[\frac{9}{2}\right] = 4$. We obtain $n_1 = 6 \cdot 4 + 3 = 24 + 3 = 27$ people, as we can also observe in the following figure:



**Case 2.** We discretize the width *l*, and then cover the classroom with a triangular grid.

The number of distances on one width (column) of the rectangular shape is $\left[\frac{l}{d}\right]$. On the odd numbered columns will be $\left[\frac{l}{d}\right] + 1$ people and on the even numbered columns we will use $\left[\frac{l}{d}\right]$ people. $\left[\frac{L}{\frac{d\sqrt{3}}{2}}\right]$ represents the number of distances on one length of the rectangular surface. With $k_2 = \left[\frac{2L}{d\sqrt{3}}\right] + 1$, the number of column, we get $n_2$, the maximum number of people,

$$n_2 = k_2 \cdot \left[\frac{l}{d}\right] + \left[\frac{k_2 + 1}{2}\right]. \qquad (2)$$

We consider some particular cases:

1. For $l = 8$, $L = 14$ and $d = 1.5$, we evaluate $k_2 = \left[\frac{2L}{d\sqrt{3}}\right] + 1 = \left[\frac{28}{1.5\sqrt{3}}\right] + 1 = 11$, which is odd, and $\left[\frac{l}{d}\right] = \left[\frac{8}{1{,}5}\right] = 5$. We obtain $n_2 = 11 \cdot 5 + 6 = 55 + 6 = 61$ people.

2. Let us consider now $l = 8$, $L = 14$ and $d = 2$. Then, $k_2 = \left[\frac{2L}{d\sqrt{3}}\right] + 1 = \left[\frac{28}{2\sqrt{3}}\right] + 1 = 9$, which is odd, and $\left[\frac{l}{d}\right] = \left[\frac{8}{2}\right] = 4$. Thus, $n_2 = 9 \cdot 4 + 5 = 36 + 5 = 41$ people.

Taking into consideration the above results, we suggest that the optimal arrangement of the students and their teacher is obtained using equilateral triangles, in one of the two cases that we have studied.

**<u>Conclusion:</u>** Based on the previous calculations, the optimal number of the people sitting in a $L \times l$ rectangular classroom is the maximum value of the values obtained in (1) and (2), that is:

$$n = \max\{n_1, n_2\}$$

$$= \max\left\{k_1 \cdot \left[\frac{\boldsymbol{L}}{\boldsymbol{d}}\right] + \left[\frac{k_1 + \mathbf{1}}{\mathbf{2}}\right],\ k_2 \cdot \left[\frac{\boldsymbol{l}}{\boldsymbol{d}}\right] + \left[\frac{k_2 + \mathbf{1}}{\mathbf{2}}\right]\right\},$$

where

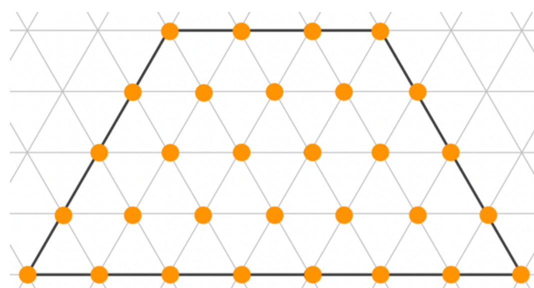
$$k_1 = \left[\frac{2l}{d\sqrt{3}}\right] + 1 \text{ and } k_2 = \left[\frac{2L}{d\sqrt{3}}\right] + 1.$$

Here, $[x]$ denotes the integer part of $x$ (the floor function) (13).

***For a class of 30 students, what would be the minimum requirements for a classroom that would accommodate everyone and maintain social distance?***

Considering the placement of 30 students, the representation shown below is the most efficient way to arrange them in the classroom, while keeping the social distance. For this arrangement, we used the equilateral triangles method, and we made the representation of the surface in GeoGebra app. We observe that the surface has the form of an isosceles trapezoid (14).

We denote by:
d – the length of the social distance
B – the larger base of the trapezoid
b – the smaller base of the trapezoid
H – the height of the trapezoid
$h_\Delta$- the height of an equilateral triangle



B = 7 · d $m$

b = 3 · d $m$

$h_\Delta = \frac{d\sqrt{3}}{2}\, m$

$H = 4 \cdot h_\Delta = 4 \cdot \frac{d\sqrt{3}}{2}\, m = 2d\sqrt{3}\, m$

$\mathcal{A} = \frac{(b + B) \cdot H}{2} = \frac{(7 \cdot d + 3 \cdot d) \cdot 2d\sqrt{3}}{2} = \frac{10\,d \cdot 2d\sqrt{3}}{2} = 10d^2\sqrt{3}\, m^2$

Let us test the above formula for some particular cases:

**Case 1**: if we consider d=1.5*m*
$\mathcal{A}= 10d^2\sqrt{3}\ m^2$
$\Rightarrow\mathcal{A}= 10 \cdot 1.5^2 \cdot \sqrt{3} = 10 \cdot 2.25 \cdot \sqrt{3} \cong 39\ m^2$

**Case 2** : if we consider d=2*m*
$\mathcal{A}= 10d^2\sqrt{3}\ m^2$
$\Rightarrow\mathcal{A}= 10 \cdot 2^2 \cdot \sqrt{3} = 10 \cdot 4 \cdot \sqrt{3} \cong 69\ m^2$

So, the minimum requirements for a classroom that would accommodate everyone and maintain social distance would be to have the area equal to $10d^2\sqrt{3}\ m^2$.

## *Conclusions*

At first, we thought it would be easy to arrange the students in a classroom, while keeping some distance between them. When we actually started solving the problem, we realized that it wasn't as easy as it seemed when we first read it. However, what motivated us was that it was describing a situation we have been facing since March 2020. We realized that our teachers at school had to do the same arrangement in our classroom, in order to stay safe when we were going to school. After we have finished solving the problem, we have seen that it was just about arranging people in the shapes of geometric figures and, finally, we obtained the desired arrangement, which included the maximum possible number of people.

## Editing Notes

(1) The use of such shapes is not justified. Admittedly the chosen shapes (triangles, squares, hexagons) are the only regular polygons that can fill the plane without superimposition. But how to make the correspondance between the shapes and the dots representing people? This is not clear. Circles (disks) alone make sense (with one people at each center), since filling the plane with non-intersecting disks of equal radius prevents their centers to be too close to each other.

(2) The best coverage of the tested configurations is not necessarily the best coverage. Indeed all the configurations are not tested. The given solution cannot pretend to be optimal.

(3) The filling is always started at a corner of the rectangle to maximize the number of people.

(4) One could also imagine a filling that would not be aligned with any of the sides of the rectangle. At least for this example it seems that this would deteriorate the capacity of the rectangle.

(5) This option is clearly suboptimal since each area outside the disks could contain one people and still maintain the social distancing. The placing of the dots (representing people) from the circles is questionable.

(6) Once more, the placement of the dots from the hexagons is not clearly justified. Indeed, the resulting configuration is the same as in option 2, so the use of hexagons versus triangles is not justified. Comparing to option 2, the alignment on the bottom side would be another option (and

the non-alignment too).

(7) Why not imagine other options with irregular placements?

(8) Same remarks as in the case $d = 1.5$ m.

(9) There are cases where the triangular option is not optimal (for example take $l = L = 1$ and $d = 1$: the square option is better).

(10) The assumption $l < L$ is never used. This assumption, together with case 2 (page 8) (which exchanges the roles of $l$ and $L$), could have been dropped.

(11) The triangles are aligned on the side of length $L$. If the upper figure is considered with the side of length $L$ being horizontal, one alignment makes a row (and not a column as it is written afterwards). Moreover, using the integer part ($[x]$, the floor function) is awkward (and makes the results untrue). The good tool is the *upper* integer part (which can be denoted $[[x]]$), also called the ceil function, which is the integer just above $x$. Indeed, if the length $L$ is a multiple of $d$, then the number of people should not be $[L/d] + 1$ but $L/d$. So all the calculations are correct only for $L/d$ non integer.

(12) 38 people is not the optimal result ; but the optimal result will be derived just afterwards in Case 2, turning the rectangle by 90°.

(13) This results are not true in full generality. First the case where $d$ is a multiple of $l$ or $L$ is not properly handled. But not only :

- consider the case $d = 2$ with a rectangle of size 1,5x1,5 : only 2 persons could be put inside (diagonal), whereas the formula gives $k_1 = k_2 = 1$ and $n = 3$.

- consider the case $d = 2$ with a rectangle of size 2,01x2,01: 4 persons can be put insize (square configuration), while the formula gives $k_1 = k_2 = 2$ and $n = 3$.

More generally, when $d$ has the same order of magnitude as one of the lengths $l$ and $L$, the optimal configuration is often irregular. And conversely when $d$ is much smaller than $l$ and $L$, then the borders of the classroom become more and more insensitive and the optimal configuration is close to the triangular one (optimal planar density on the infinite plane) and the proposed formula becomes more accurate.

(14) Indeed, the trapezoid is one of the admissible convex shapes that minimizes the surface, but not the only one ; another optimal configuration is the parallelogram with sides of lengths 5 and 4 and angle 60°.

But other shapes achieve even smaller surfaces. In this example, an equilateral triangle with side length 6 can contain 28 people; 2 small equilateral triangles with side 1 aggregated to this big triangle give space to 2 more people and yields a (non convex) shape of area $(9 + 1/2)\sqrt{3}$ which is smaller $10\sqrt{3}$, the area of the trapezoid. Other shapes can achieve even lower areas, up to 0 (infinitely thin rectangles for instance), which shows that the problem is complex and needs some further assumptions to be adequately solved.