Bots on grid

1. Presentation of the research topic.

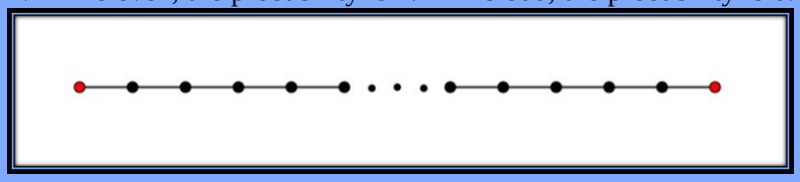
Given a (x,y,z) three-dimensional grid and two different bots moving one unit per step, towards the opposite corner, at the same pace, starting at the same time, from points (0,0,0), respectively (x,y,z), we try to find chances of the two bots meeting on one of the grid's nodes.

2. Brief presentation of the conjectures and results obtained.

Two bots walk on a three-dimensional integer grid. One bot starts from (0,0,0), the other bot starts from (3,4,5). In one step, each bot can walk only one unit, at the same pace, in either direction. All trajectories are considered to have equal probabilities. The first bot can only increase its coordinate, while the second bot can only decrease its coordinate. What are the chances that the two bots meet on a node of the grid? What if the movement's direction of the robots is chosen with equal probability at each step? Generalize the problem if possible.

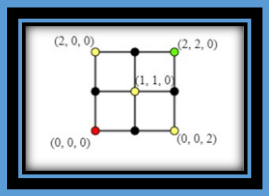
Case I. A segment.

If two dimensions are null, the parallelepiped is a segment of length x. If x is even, the probability is 1. If x is odd, the probability is 0.



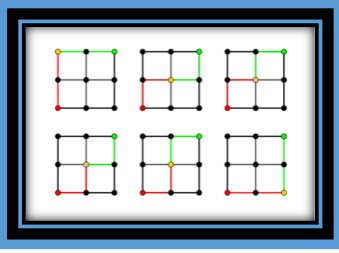
Case II. A 2×2 square.

One robot starts at the red point, one starts at the green point, they can only meet in the yellow points.



There are 6 favorable cases and 16 possible cases.

The probability is *p=37.5%.*



Case III. The parity of the sum of the coordinates (S).

If S is odd, the robots never meet. If S is even, the robots meet at least once.

Case IV. A 3×4×5 parallelepiped.

In order to solve this case, we write all possible movement sequences and compute their probabilities. By multiplying the symmetrical probabilities, we find the final probability. Example: the following sequences each have the probability $(\frac{1}{2})^3 \cdot (\frac{1}{3})^3 = \frac{1}{216}$:

0 0 0 1 1 1

0 0 0 1 1 2

0 0 0 1 2 2

0 0 0 2 2 2

The final probability is approximatively ***5.73%.***

4. Conclusion.

To solve this problem, we used probability and combinatorics. We analyzed some basic particular cases: the case in which the grid is a segment and the case in which the grid is a 2×2 square. We have proved that the bots only meet if the sum of the dimensions is even, otherwise they will pass not. Then, we solved the case when the grid has coordinates *(3, 4, 5)*, obtaining a probability of ~5.73%.

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