Colegiul Național "Emil Racoviță",
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Evolution of parasites

Topic 9

The problem we had to solve

In an isolated environment, we study the relation between a certain type of parasite and their host and how these evolve with time t (continuous or discrete). In our model, parasites deposit eggs on their hosts and when the eggs hatch, the host dies. Denote by H and P the number of hosts and parasites respectively (these can be modelled as a function of t). At each step (unit time), the number of eggs deposited depend on the probability that a parasite and a host meet. One can assume that this probability is proportional to the product H.P of the populations. We are given fixed values b and d for the birth and death rate of hosts when no parasites are present.
Moreover, we let dp be the death rate of the parasites.
Run simulations for given values of b, d and dp and try to determine what happens with the populations H and P in time.

As we started to work on the problem, we decided to do the experiment ourselves. We went to the USAMV (a university here in Cluj) lab and we worked on **3 hosts** and **310 parasites**. The parasite we used is called *Strongylus equinus* and the experiment lasted for 10 days. In **3 days**, **122 eggs died** and **188 remained**. In the next week, another **30 died** and **158 eggs hatched**. While working on the experiment, we found various data about parasites and their hosts, and we discovered that once the host is infected with a parasite, it will show clinical symptoms in around 8-9 months. However, most of the hosts don't die after being infected with the parasite.

Final formula

Our final formula is:

$$P_t = k \times (e^{-d_p} \times t + c)$$

P_t is the parasite population at time *t*

k =1000 (constant)

e^{-d_p} = 0.8

d_p = -ln(0.8)

d_p =parasite death rate

c =variable

Conclusions

But before we discovered the final formula, we had several other attempts at equations and equalities that eventually led us to the desired result. $d_p = P_{(t+1)} - P_{(t)}$

$$P_{(t+1)} = P_{(1)} + t \times c \xrightarrow{\text{example}} P_{(100)} = P_{(1)} + 99 \times c$$

c is a variable whose values may differ depending on the problem data. From the two formulas above we extracted that $d_p = t \times c$ but we needed more to advance.
We also arrived at the formula: $P_{(t+1)} = P_{(t)} + (b_r - d_p) - t \times c$ but we don't think it's very correct, so we didn't dwell on it too much.

The biggest discovery we made was that the problem was solely based on something theoretical, not on biological verified data.

The Lotka-Volterra system

By considering only the two populations and neglecting the hatching period, a non-linear system of differential equations that describe the evolution of the populations can be written:

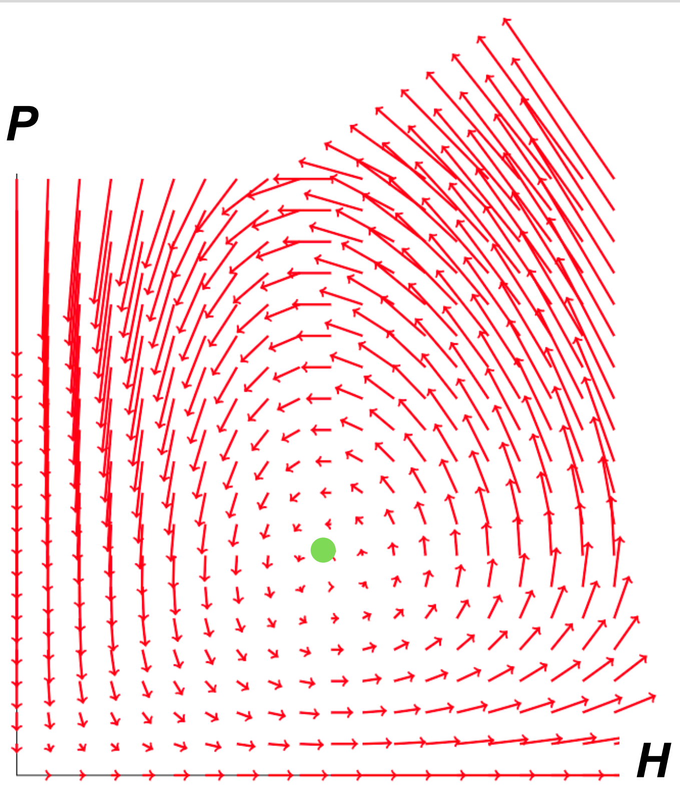
$$\begin{cases} \frac{dH}{dt} = (b-d)H - k_1 HP \\ \frac{dP}{dt} = -d_P P + k_2 HP \end{cases}$$

where: H = H(t) the population of hosts in time and P = P(t) the population of parasites in time

Vector fields

The Lotka-Volterra system is not solvable but it gives us explicit values for the derivatives of the two populations for any given pair of populations (H, P), i.e. the *direction* in which each population evolves from then on. This can be represented on a plane in which the populations are the axes, each point being assigned a vector indicated by the system.
There is just one non-zero point at which the vector is zero - that is a point at which the populations remain constant in time. It is (green on the figure) located at

$$\left(\frac{d_P}{k_2}, \frac{b-d}{k_1}\right)$$

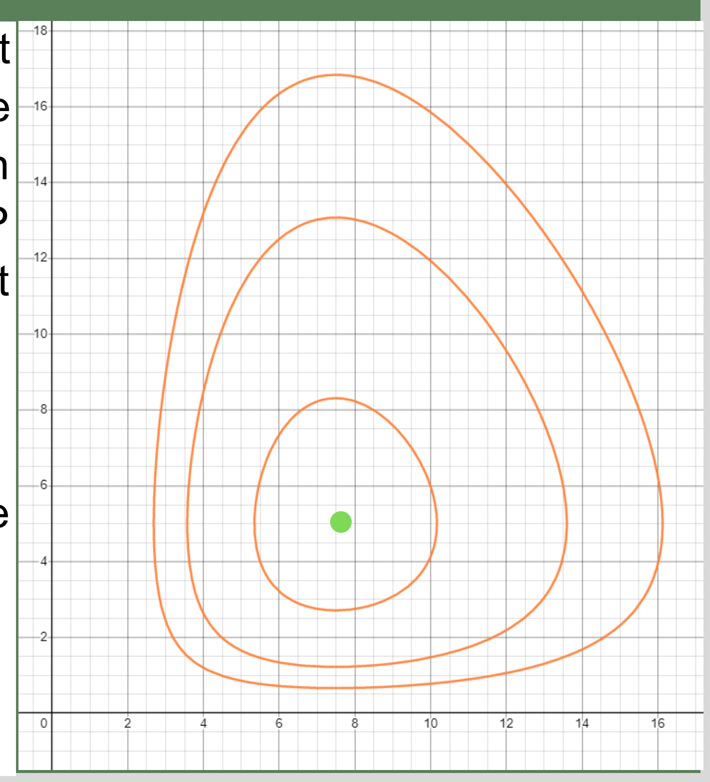


The prime integral of the system

The Lotka-Volterra system cannot be solved explicitly, however, by the method of variable separation, an implicit curve depending on H and P is obtained. It is a closed path that cycles around the stationary point.

$$(b-d)\ln y - k_1 y + d_P \ln x - k_2 x = C$$

where C is a constant indicating the distance from the path to the curve



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