``ENI'' - Game and Probability

Two players: H *(the **h**iding)* and S *(the **s**earching)* play the following "ENI" - game. Before starting they choose the size *n* of the board ($3 \leq n$ and let's agree that $n \leq 10$ here). For the "PLUS" - version, they also choose number *k* of the black stones, where *k=1,..., n-2* (for the basic version of the game *k* is just fixed to be 1).

So, the board consists of *n* boxes with the numbers *1, 2, ..., n* − these are the places, where H can hide his treasure. To do this, H has *n* "golden" tiles, with numbers *1, 2, ..., n* on the back side, and he chooses one of them − this is the place where the treasure is hidden. He also has 1 black stone (or *k* black stones, in the "PLUS" version).

First H puts the chosen golden tile next to the board, golden side up, without showing its number to S. Now S tries to find the treasure: he has a white stone to indicate it, and he puts his stone on the board at the number of his ``first guess".

This is the moment, when H uses his black stone(s) − he puts the black stone (or all *k* black stones, for "PLUS" version) on the board, but − ATTENTION! − now he can occupy only the numbers belonging to **the ENI** set: the set of all places **E**mpty (= without the treasure) and **N**on-**I**ndicated (by the white stone). Now S should decide: he can either change his first guess or not. If he decides to change it, he moves his white stone onto the place of his "second choice" (it is forbidden to put it on any place occupied by black stone).

Now the players check the number on the bottom side of the golden tile initially chosen by H. If the number is equal to that indicated by white stone − S wins. In the opposite case: H wins. This is the end of the round, and the players play the second round with changed roles H and S, but with the same value of *n* (and of *k*, for the "PLUS" - version) − this value(s) can be changed only after each two rounds.

The goal of this Exercise/Subject is a probabilistic explanation of the results of the "ENI"-Game, after large number of tests. Especially, we want to test the influence of changing\non-changing the first guess of S.

So, in **the first part of the exercise** you should play a lot of rounds, with many parameters n (and k) and with the both types of the decisions of S. And the players should **always note down all the results of the rounds**. So, each time, please note down:

- name (or, e.g., a unique one-letter nick name) of H and of S
- the parameter(s) *n* (and *k*)
- has S changed his first guess?
- has the treasure been found?

In **the second part of the exercise** you will compute (separately for each parameter *(n* and *k*)) the "experimental probability" of finding of the treasure, i.e., this probability, which follows from the results of your tests/plays with the ENI-game, for **two cases**:

a) when the first guess of S was NOT changed,

b) when the first guess of S was changed.

In **the third part of the exercise** you will try to find the theoretical explanation of the above computation, and find the "theoretical probability" for the both cases and different parameters.