Sotto la pioggia - Sous la pluie

To walk or to run under the rain?

2018 - 2019

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How many times has it happened to you that it started to rain out of the blue while you didn't have an umbrella? Or that you had an important rendez-vous and you arrived wet from head to toe? At this point, we've all thought about how we could have gotten less wet, for example by changing our speed. Hence, we decided to look for a mathematical answer to this problem, starting by answering the question: is it better to walk or to run when it rains?

This article is a part of the presentation made in Iași MATh.en.JEANS congress about the problem "it's raining! Is it better to walk or to run?", a group work between the Lycée Stendhal of Milano and the Liceo Bruni of Padova. It shows the mathematical physical aspect. Others aspects as the probabilistic point of view or from the droplet point of view are not described in this article.

PRACTICAL SIMULATION

First of all, we analyzed the situation from a general point of view by creating an algorithm that calculates the number of droplets that touches a person depending on the intensity of the rain (we considered for the algorithm a windless rain) and on the speed of the person. This enabled us to have experimentally an idea about what the conclusion could be and how the quantity of water changed. The conclusion that we could draw was that the more the speed of the person increased, the less the person was getting wet.

Video of the algorithm†

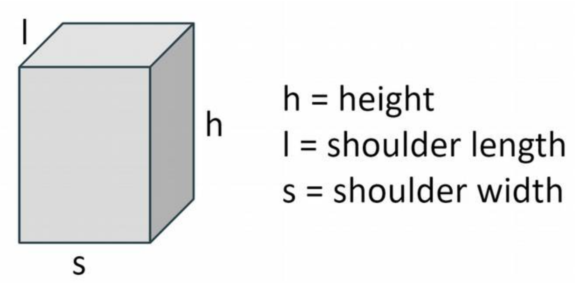
This video shows the functioning of the algorithm. The red rectangle represents a person and the blue dots represent the droplets of rain. The algorithm calculates the number of droplets (the number appears at the bottom left) that touch the person when the person goes from one side to another. It also permits to simulate different situations, for example by changing the speed of the person or the intensity of the rain. In the video, we increased each time the speed of the rectangle. We can see that the number of droplets decreases.

† *Video available on **Vimeo**: https://vimeo.com/573627983*

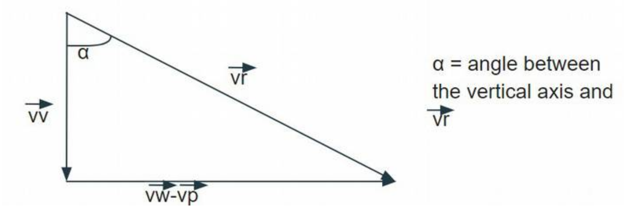
MODELLING PROCESS

Later, we decided to take into consideration the case when the rain is subject to a constant frontal wind (1). We chose to represent mathematically the person as a 3D figure with a prism and the rain as the vector $\vec{v}_r$ This vector $\vec{v}_r$ has two elements. First, a vertical component which represents the rain falling vertically (we called it $\vec{v}_v$) and second a horizontal one which represents the wind (we called it $\vec{v}_w$). Since we are considering a person who is moving, walking or running, we shall represent this movement by the vector $\vec{v}_p$. We then have:

$$\vec{v}_r = \vec{v}_v + \vec{v}_w - \vec{v}_p . \quad (2)$$



The person in three dimensions



Representation of the vector $\vec{v}_r$

TAKING INTO ACCOUNT DIFFERENT TYPES OF RAIN

Since there are various types of rain, we decided to take three different cases into consideration: when it is drizzling, when it rains normally and when it pours. Then we compared information coming from different internet sources about the key numbers corresponding to these types of rain and did some measurements ourselves. We considered that the average volume of a droplet is $0{,}05\ \text{cm}^3$ and that the average surface of the top of the head is $250\ \text{cm}^2$ and of the shoulders is $2\times 130\ \text{cm}^2$ for a total of $510\ \text{cm}^2$ for the upper part of a person. This table shows numbers relative to a vertical rain falling in the absence of wind.

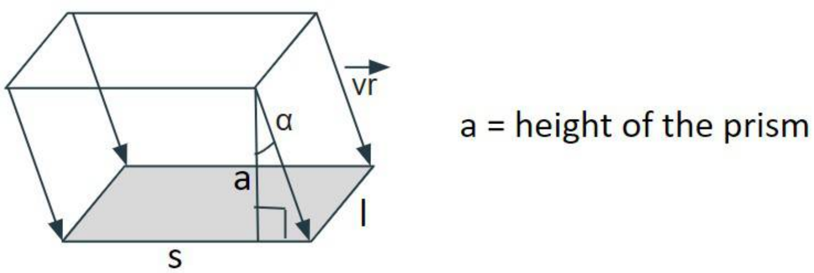
According to some weather forecast sites, these are the statistics related to each case for a person standing still in the rain:

	when it is drizzling	when it rains	when it pours
average quantity of rain on 1 cm^2 each second	$0{,}025$ L	$0{,}1$ L	$0{,}25$ L
average quantity of water touching a person each second	$1{,}27\times10^{-3}$ L	$5{,}08\times10^{-3}$ L	$13{,}12\times10^{-3}$ L
number of droplets touching the upper part of a person each second	25	25	260
speed of the droplets (when it rains more intensely the droplets fall quicker)	2 m/s	6 m/s	9 m/s
number of droplets per unit volume (p)	18 droplets/m^3/s	78 droplets/m^3/s	202 droplets/m^3/s

THE CALCULUS (1): THE UPPER PART OF THE HEAD

When someone goes under the rain, the main parts of the body that are touched are the upper head and one of the sides of the body (the actual side depends on the wind direction). We decided to calculate separately the quantity of water that touches the upper part of the head and the quantity of water that touches the side of the body and then to sum them up.

Regarding the upper part of the head we can summarize the situation as follows:



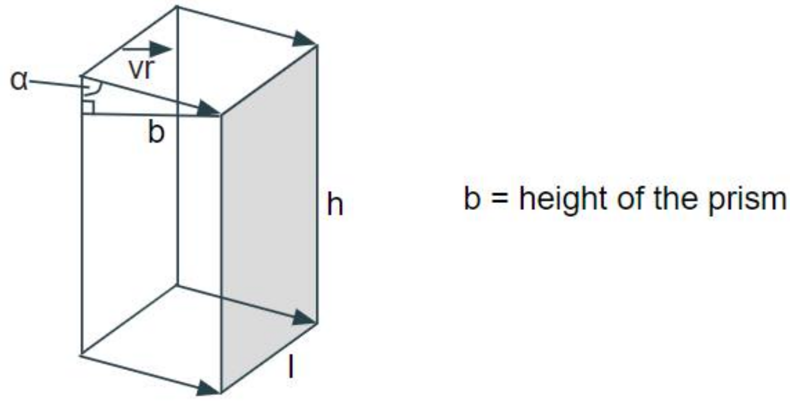
In one second, the rain goes $v_r=\|\vec{v}_r\|$ meters (3). Hence, each second, the head comes into contact with a volume $V=I\times s\times a$ of water.
According to the trigonometric formula, $\cos(\alpha)=a/v_r$ i.e. $a=\cos(\alpha)\times v_r$. If we come back to the representation of the vector $\vec{v}_r$ we can see that $\cos(\alpha)=v_v/v_r$. Therefore, if we replace $\cos(\alpha)$ by v_v/v_r in the first expression, we have $a=v_v$ and so $V=I\times s\times v_v$. Then, since the whole prism

is not full of water, we need to multiply V by the density p. Moreover, we need to multiply V by the exposure time $t=d/v_p$.
As a consequence, the quantity of water that touches the upper head equals to:

$$Q_{\text{head}}=I\times s\times v_v\times p\times d/v_p,$$

THE CALCULUS (2): THE SIDE OF THE BODY

Regarding the side of the body we can summarize the situation as follows:



The calculus is really similar to the previous one. In fact, we can see that, each second, the body comes into contact with the volume of water $V'=I\times h\times b$. According to the trigonometric formulae, $|\sin(\alpha)|=b/v_r$. If we come back to the representation of the vector $\vec{v_r}$ we can see that $v_w-v_p=\sin(\alpha)\times v_r$. Therefore, if we replace $|\sin(\alpha)|$ by b/vr in the last expression, we have $b=|v_w-v_p|$ and so $V'=I\times h\times|v_w-v_p|$. Then, since the whole prism is not full of water, we need to multiply V' by the density p. Moreover, we need to multiply V' by the exposure time $t=d/v_p$.
As a consequence, the quantity of water that touches the side of the body equals to:

$$Q_{\text{body}}=I\times h\times|v_w-v_p|\times p\times d/v_p.$$

Hence, the quantity of water Q that touches the person is equal to $Q=Q_{\text{head}}+Q_{\text{body}}$,

$$Q=I\times s\times v_v\times p\times d/v_p+I\times h\times|v_w-v_p|\times p\times d/v_p$$
$$Q=I\times p\times d(s\times v_v/v_p+h\times|v_w-v_p|/v_p)$$
$$Q=I\times p\times d(s\times v_v/v_p+|h\times v_w/v_p-h|)$$

Despite the obvious simplifications we had to make for modeling purposes, this formula shows that the quantity of water that touches a person is related to the speed of the wind. Moreover, it is interesting to see that a person should move as close as possible to wind's speed, in order to eliminate Q_{body} (4): indeed, when $v_p=v_w$, Q_{body} is equal to 0. This means that you should adapt your speed to the situation, therefore the only thing that is needed not to get too wet is adaptability and a portable anemometer in your pocket – bear in mind though that an umbrella might be cheaper.

COMPLEMENT ABOUT Q

Average human shoulder width for a man $s=41$ cm (from: https://www.healthline.com/health/average-shoulder-width).
Average human height for a man $h=171$ cm (from: https://ourworldindata.org/human-height).
So, $s/h \approx 0{,}24$.

We considered 3 cases for v_v (2 m/s, 6 m/s and 9 m/s), depending on the speed of the rain.
Let's take the "pouring" example (so the one with the highest v_v):

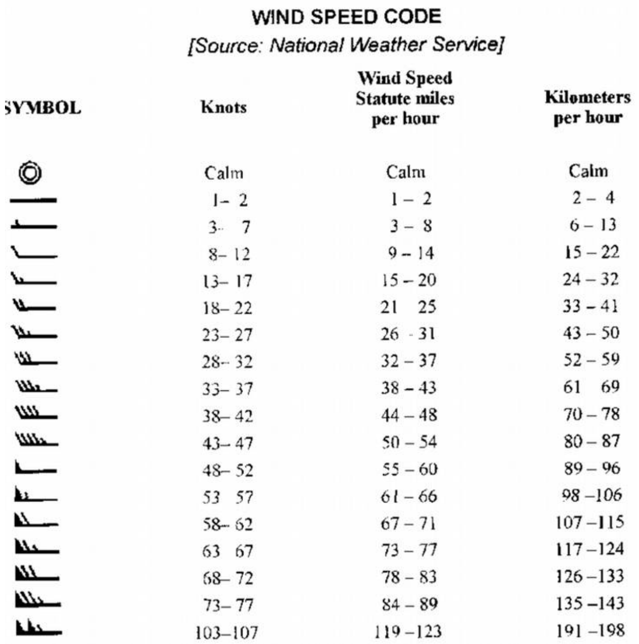
If $\frac{v_w}{v_v} < \frac{s}{h}$, then $v_w < \frac{s}{h} \times v_v$ (since v_v is always positive)

Therefore, $v_w < 0{,}24 \times 9 < 2{,}2$ m/s $\approx 7{,}9$ km/h

If we consider the other cases, we get:

- In the "drizzling" case: $v_w < 0{,}24 \times 2 = 0{,}48$ m/s $\approx 1{,}7$ km/h
- In the "raining" case: $v_w < 0{,}24 \times 6 < 1{,}4$ m/s ≈ 5 km/h

As we can see in the "National Weather Service" chart, this kind of wind speed is, even at its maximum, considered almost insignificant (and 7,9 km/h was obtained with the highest v_v: the "drizzling" example gives us 1,7 km/h, which is completely insignificant) .



WIND SPEED CODE
[Source: National Weather Service]

SYMBOL	Knots	Wind Speed Statute miles per hour	Kilometers per hour
	Calm	Calm	Calm
	1– 2	1 – 2	2 – 4
	3– 7	3 – 8	6 – 13
	8– 12	9 – 14	15 – 22
	13– 17	15 – 20	24 – 32
	18– 22	21 25	33 – 41
	23– 27	26 - 31	43 – 50
	28– 32	32 – 37	52 – 59
	33– 37	38 – 43	61 69
	38– 42	44 – 48	70 – 78
	43– 47	50 – 54	80 – 87
	48– 52	55 – 60	89 – 96
	53 57	61 – 66	98 –106
	58– 62	67 – 71	107 –115
	63 67	73 – 77	117 –124
	68– 72	78 – 83	126 –133
	73– 77	84 – 89	135 –143
	103–107	119 –123	191 –198

Therefore, when $\frac{v_w}{v_v} < \frac{s}{h}$, we can return to our first case, in which, as the algorithm showed us, in absence of wind the number of droplets decreases when the person's speed increases.

Hence, we can say that until v_w is "insignificant", so inferior to this maximum value that allows $\frac{v_w}{v_v} < \frac{s}{h}$ (so, 7,9, 1,7 or 5, depending on the speed at which the droplets are falling), Q is decreasing and the fastest you run, the least wet you will get.

As soon as v_w gets more "significant", which we can define as $\dfrac{v_w}{v_v} \geq \dfrac{s}{h}$, then it allows a minimum (when $v_p = v_w$), which gives you the optimal speed you should move at.

EDITING NOTES

(1) Wind velocity is assumed to be co-linear to the person's movement. It can be oriented in the same direction (tail wind – the most interesting case) or in the opposite direction (head wind).

(2) Here, the rain velocity $\vec{v}_r$ is *relative* to the person in motion.

(3) It will be also noted $v_v = \|\vec{v}_v\|$ and $v_p = \|\vec{v}_p\|$. For wind velocity, the horizontal line of the movement being oriented according to $\vec{v}_p$, we have to let $v_w = \|\vec{v}_w\|$ in case of a tail wind and $v_w = -\|\vec{v}_w\|$ in the contrary case, so that $|v_w - v_p| = \|\vec{v}_w - v_p\|$ in both cases and then the computation for the side of the body remains correct.

(4) This is not always the case, as stated at the end of the article (next section).
Indeed, when $v_p \leq v_w$ the formula for Q becomes $Q = I\,pd[(s\,v_v + h\,v_w)/v_p - h]$ and it is clearly a decreasing function of v_p, so the minimum on $(0, v_w]$ is taken for $v_p = v_w$. But for $v_p > v_w$, we have $Q = I\,pd[(s\,v_v - h\,v_w)/v_p + h]$ and the direction of variation depends on the sign of $s\,v_v - h\,v_w$. If $s\,v_v - h\,v_w \leq 0$, that is $v_w/v_v \geq s/h$, the function $v_p \rightarrow Q$ is non-decreasing on the interval $[v_w, +\infty)$, so we have actually a minimum for $v_p = v_w$.
On the contrary, if $v_w/v_v < s/h$ and so $s\,v_v - h\,v_w > 0$, the function $v_p \rightarrow Q$ is decreasing on $(0, +\infty)$, and the person has to move as fast as possible – note that this is always the case if we have a head wind $(v_w < 0)$.