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Gravity Calculator

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Crișan Anda, Hangan Miruna, Iuga Raul, Gherman Vlad, Lăpușan Alexia, Pop Rucsandra - 10th grade

Dobra Gloria, Friciu Mara, Hedeșiu Petra - 11th grade

School(s) : Colegiul Național “ Emil Racoviță”, Lycee Val De Durance

Teacher(s) : Văcărețu Ariana

Researcher(s) : Yves Papegay, INRIA - Sophia Antipolis Mediterranee Research Center

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1. Introduction

1.1. The problem

We need to code two numbers with beads and then make a circuit so that when all the beads are released, we obtain the result of the sum of the two numbers.

1.2. Results

We've managed to solve the problem for various numeration bases, including binary base and decimal base. The problem could've been approached through different ways and in different numeration bases. More than just the theoretical approach, we've built a physical system which manages to prove what we've discovered through the theoretical approach.

2. The Solutions

2.1. Base 10 solutions

2.1.1 First team's base 10 solution

We've established a strict language so that we can address the problem terminology through an easier process. Working in the decimal base (base 10) we need to be familiar with a set of specific terms we will call Z the tens balls and U the units balls, we'll name the Z tube the tens tube and the U tube the units tube (fig. 1). More than this we will use the so called "L Gates" which are a tool which helped us to automatizate the system.

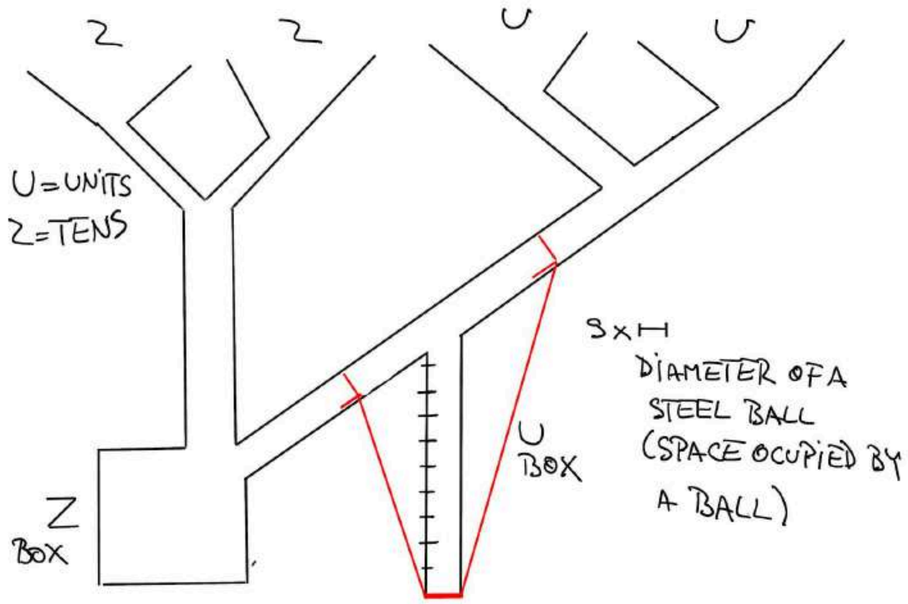


Figure 1: "Base 10 system"

We've begun by approaching the problem through different ways, until we've found the most easier and practical one. The topic stated that our solution mustn't include the physical action of a human, so it will need to be fully automatizated. The most efficient way was by groping the units balls and the tens balls into their own tubes, the U tube and the Z tube, which will contain the final amount of each kind of balls (U and Z) after we've released the corresponding number coded in the 10 base. This was the first most efficient step in building our initial approach to the problem.

The second step involved adding a lot of different numbers coded in the decimal base and hoping to find a similarity between them, which will lead us to a rule or principle. After many examples and exercises we've identified a principle (rule) which applied in all the cases we've chosen to take as an example. The following rule was identified, similar to the basic principle of addition, when the units balls exceeded more than 9 balls, the tenth one needed to transfer, move on to the tens tube and will count as a tens ball not as an unit ball. Like when adding two numbers if two digits added exceed 9 we need to retain a ten in mind and add it to the next set of digits. For this to be possible as well in the physical representation of the system, we needed to to create a system which would be able to do it fully automatized. So we began by creating a unit tube which only fits 9 balls and from there on each tenth unit ball in any case needs to fall in the tens tube. We've had only a little problem with the fact that if the total number of units balls exceed 10 and we will have more balls that need to fit the compartment which only fits 9 balls, that would've meant that all the remaining units balls would fall in the tens tube and that wouldn't be correct. So we've thought about creating a system of "L gates" that are interconnected and control a gate which closes and opens the unit tube and allows the balls to leave the system. These two gates have a precise initial position and a tight rope which connects them to the simple gate.

We need to clarify the initial and final position of the two "L gates" and the regular gate. Starting from the initial position (fig. 2) the gates move to the final position (fig. 3) and then return to the initial position when the tenth unit ball comes through the system. The units final compartment is placed between the two "L gates" and has a regular gate at the bottom which can move itself. Now that we've clarified our general view of the system, we've tried to realise a suited design and we've begun to draw the whole system as a one, containing all components. The two entrances for the tens' balls and the two from the units' balls, the tens' tube and the units' tube, and the two final compartments.

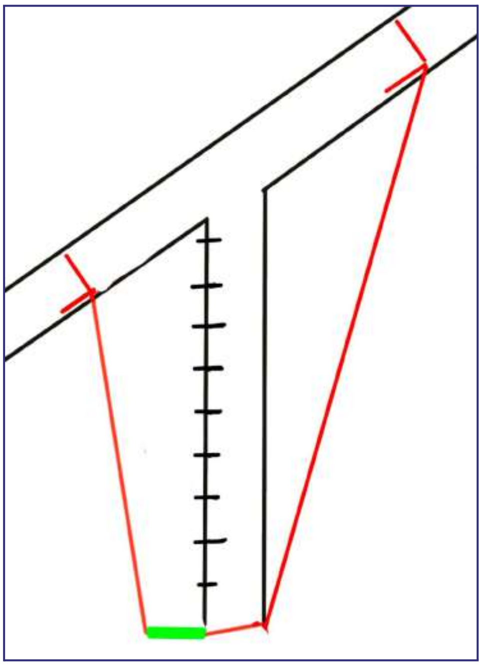


Figure 2: "Initial position"

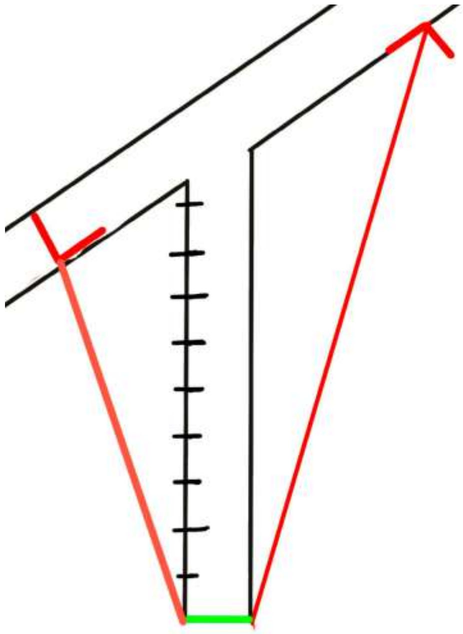


Figure 3: " Final position"

We'll begin by explaining step by step how the balls fall through the system and how the gates move around depending on the balls. First we'll begin by entering the first unit ball in the system. The ball will hit at first the first "L gate" which will rotate and allow the ball to fall down in the units compartment, simultaneously, the second " L gate" will come up and place itself in the final position in which it will be hit by the 10'th unit ball, and at the same time the regular gate will close so that it will allow the unit balls to gather in the compartment. After the other eight unit balls fall in the compartment, the 10'th one will jump over the tube and will continue its path down the unit tube. It will jump over because it will hit the 9 th unit ball which stands at the peak of the compartment. Whe this ball pushes the second " L gate " the first " L gate " will return to its initial position and the regular gate will open, therefore the 9 unit balls which are in the compartment will exit the system (fall out the system) and the tube will empty itself. From here the 10 th unit ball arrives in the tens compartment and will count as a tens ball not a unit ball anymore. Then, when the 11 th ball falls down the system, it will push the first " L gate " and will place it in the final position, as well as the other " L gate " and the regular gate. And the rest of the balls will gather in the unit tubes and only the remaining batch of balls will be taken into consideration at the end. The tens process is simple, we only introduce the corresponding number of tens ball in the system and they will all fall in the same place, the tens compartment.

The system works in the same way for all kinds of addition, only with tens or with hundreds, thousands, tens of thousands, tens of hundreds, millions and so on. The compartment which fits only 9 balls and the two " L gate " and the regular one will apply to all of them. We just multiply the test for every digit unless the one from the last position, from the tenths, the tenths one, from the hundreds, the hundreds one; from thousand, the thousands one; from tens of thousands, the tens of thousands; from tens of hundreds, the tens of hundreds; from millions, the millions.

*To add we number the tubes from left to right and we read the coded number in the end from down to bottom

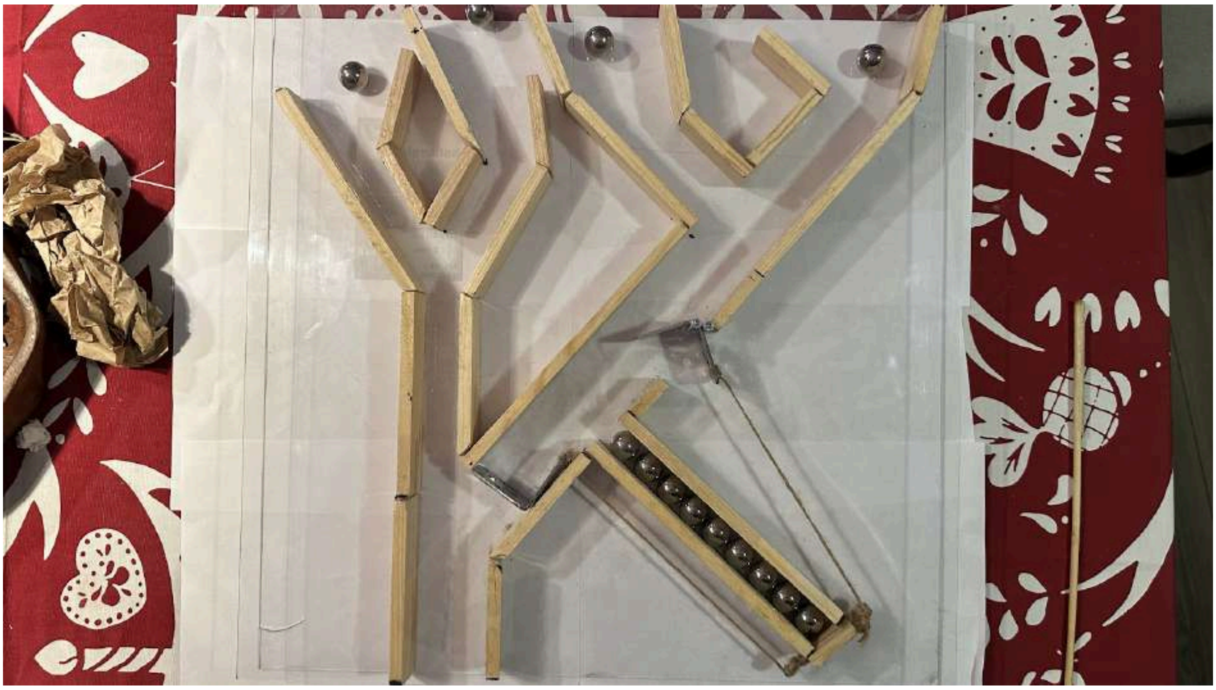


Figure 4: "*Physical system"*

2.1.2. Second teams base 10

Fig. 5 represents the second team's system. It is a base ten calculator and there are the same rules: no human touch is allowed after the balls enter the system.

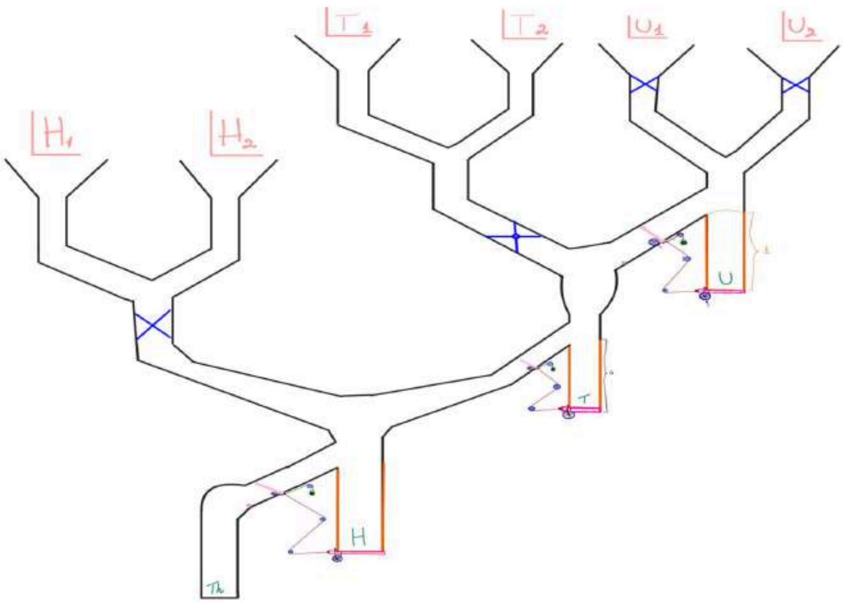


Figure 5: "Base 10 calculator"

2.1.2.1. Element's presentation

At the top of the scheme there are six burnt-like structures connected to tubes. That is the balls' entrance in the system. Each of the six has a letter marking that indicates where to throw the balls after a ten base division has been effectuated. "U" means units; "T" means tens and "H" means Hundreds and their index represents which number they come from: the first (U1,T1,H1) or the second (U2,T2,H2). Moving downwards on the scheme: to each element a number or letter has been assigned (fig. 6), except the pipes that are the general structure of the system. The element number 1 is a compartment that can only collect 9 balls. Elements represented by number 2 are wheels that help the doors move. Number 3 represents the sliding doors in question that are similar to the plaques marked with number 4, except that these do not slide, they move downwards or upwards. Connected to these doors is the fifth type of element- the weight: it is lighter than a ball, but heavier than the moving plaque. The next element is marked with number 6 and it is the rulment that is attached to a wire- number 7. Number 8 represents the speed control mechanism. Furthermore, the letters represent the compartments where balls end up (fig. 5). In the "U" marked tube, the final number of balls representing units will end up. The "T" one is the tube where the balls representing tens will end up. The next one is the "H" one where the hundreds can be counted. The last one is "Th" in which the thousands balls will end up.

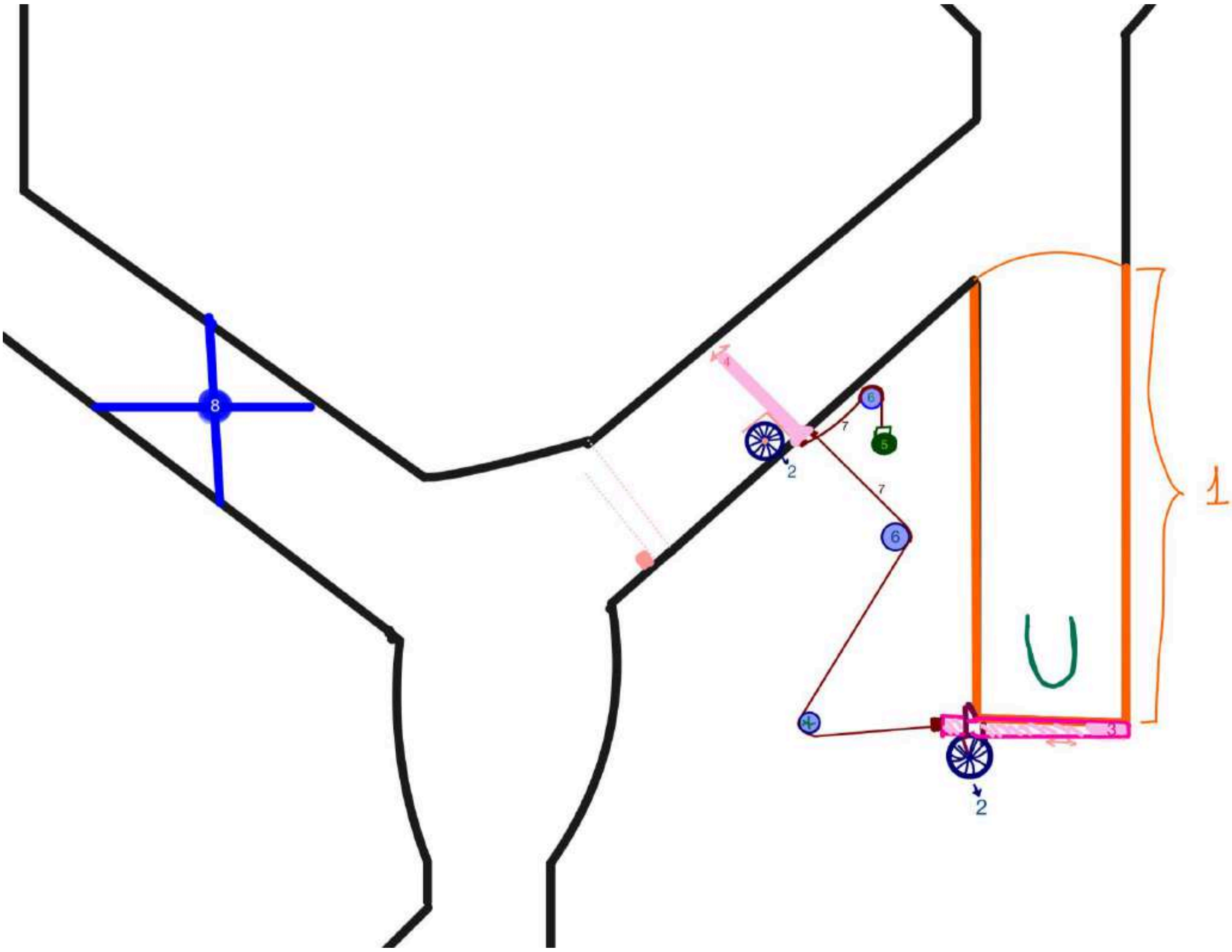


Figure 6: "The elements"

2.1.2.2. How the system works

In the creating process we tried approaching the problem in different ways, but this one seemed to be the optimal one. To be able to explain how the system works we firstly need to choose two numbers bigger than 10 and then divide them in base ten, like the following example:

First number: $59= 5* 10^1 + 9* 10^0$
Second number: $46= 4* 10^1+ 6* 10^0$

Afterwards we summed them up with the help of base-ten division:

$59+46 = (5* 10^1+ 9* 10^0)+ (4* 10^1+ 6*10^0) = 9* 10^1 + 15*10^0= 9* 10^1+ 1* 10^1+ 5*10^0= 10* 10^2 + 5* 10^0$

and we introduce the right number of units for each number. 9 balls go in the "U1" tube and 6 in the "U2" one. These balls will fall one at a time with the help of figure 8 mechanism into the container noted with the figure 1. Due to the fact that the "1" container can only fit 9 balls and the sum of 9 and 6 equals 15, the tenth ball will get directed towards the "4" platform. When it reaches that plaque, the balls weight will determine it to move downwards until the stopping point where the ball will fall into the next pipe, becoming a tens ball. Because a tenth is formed by 10 units, the first 10

balls must transform into a singular one that ends up in the tens container. In the meantime, during the platforms descent, another string bound to other 2 rulments placed in a Z position and the “3” noted sliding door, will pull on this last mentioned element. These elements working in synchronisation and the rotation of the “2”wheel will determine the “3” door to slide towards left, allowing the nine balls in the “1”container to exit the system. Those balls are no longer relevant to us. After the transformed ball falls from the “4” platform, the “5” noted element: the weight will help the plaque to move upwards until it reaches its starting point with the help of a rulment: number 6 and a string: number 7. To be noted that the platforms movement is possible due to the wheel: element number 2. After the platform is up and the sliding door has reached its initial position thanks to the wheel and a slight incline, the eleventh ball will fall in the “1”container and so will the twelfth, thirteenth, fourteenth and fifteenth which will be the balls representing the units. There will be 5 units balls. After we have watched all this process, we can introduce the tens balls like this: for the first number we introduce 5 balls in the “T1” and 4 in the “T2”. These will unite one by one, slowed down by the “8”mechanism, with the one ball already in the “T” container. The sum of 5 and 4 equals 9 and with the one ball already existent, the total is 10. Only 9 balls fit in the tens container so the tenth one that remains will fall in the hundreds tube, representing a hundred.
There is the exact same mechanism as for transforming units into tens. The 9 balls in the tens container will exit the system with the help of the sliding door “3”.

2.1.2.3. The final

In the end, the system should look like this: in the hundreds tube there is one ball, in the tens one there are none and in the units tube there are 5 balls. This translates in the result:
$1*10^2+ 0*10^1+ 5* 10^0=105$.

2.1.3 Third team’s base 10 solution

2.1.3.1 The simple system

The base case involves the summation of single-digit numbers. A system consisting of two tubes can be employed to perform this addition (fig. 7). The input balls start falling into the space meant for only nine balls (or units). If the sum exceeds 9, the container is full and the leftover balls will have nowhere else to go, but through the sideways tube. There, we have the container space in the lower section ready for another ball that will announce to us that the sum will also have balls in the tens area, not only in units. Any remaining balls will bypass the covered holes and go directly into the output units bowl. If the sum does not exceed nine, the system detects this by the absence of balls in the tens container. In this case, the gate to the first hole opens, releasing the balls into the output units bowl. This mechanism allows for the addition of two numbers, even if their sum exceeds nine, using a straightforward structure.

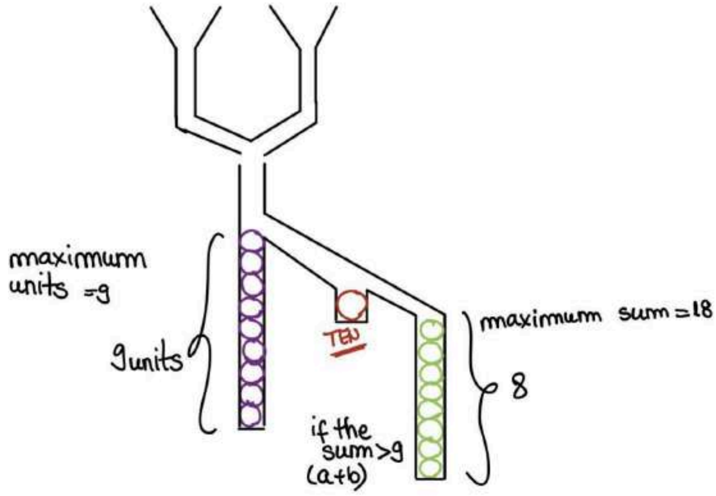


Figure 7: "Single-digit system"

2.1.3.2 Complex system for multi-digit numbers

Building on the previous simple case, the structure can be scaled to support the addition of multi-digit numbers. Fig. 8 illustrates the addition of two-digit numbers. However, a significant drawback is that the structure can become considerably large, making it difficult to construct. To address this issue, the following concepts are proposed (fig. 9)

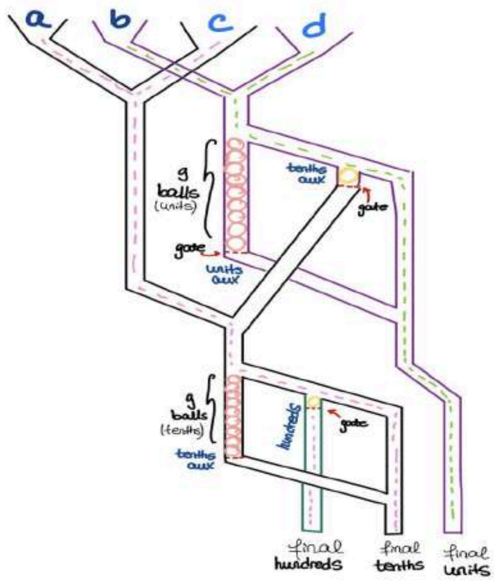


Figure 8: " Multiple digits system "

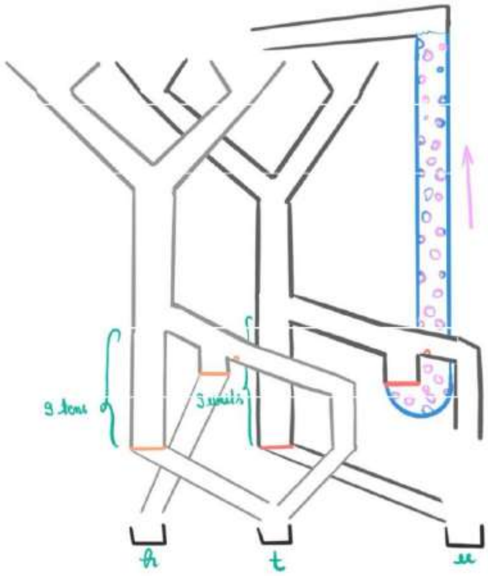


Figure 9: “Recycling-beads system”

As you can see, this new system is very similar to the previous one, the added benefit is the length, which is shortened. The way this works is by recycling the ball that ends up in the small tens container. We manage to transport it up in some way (here it is depicted with water, but the ball would need to be less dense than the liquid for this to work) and put it back in the system, but this time in the tens section (with lighter gray in fig. 9).

2.2 Base 2 solutions

After we’ve successfully managed to build the decimal system physically, we’ve started to extend our research to other numeration bases. We thought about choosing a base in which we don’t need to use so many balls. So, we’ve found the binary system in which we only work with one ball or no ball at all. So we can get rid of a lot of balls by working in the binary system.

2.2.1 First base 2 solution

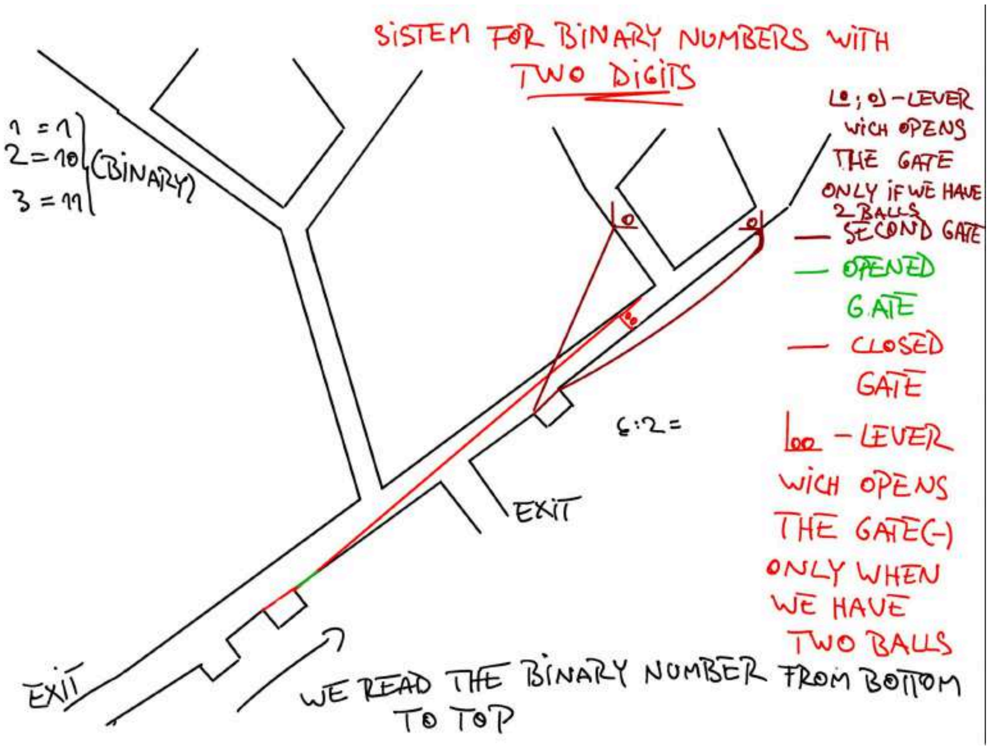


Figure 10: "Two-digit binary numbers system"

Doing so, we've tried to keep our initial way of thinking, dropping and using the same "L gates" in different ways. We began by grouping the digits one by one, byte by byte, and choosing different examples to follow the same beginning steps as we did for the decimal system. After many examples we've identified a pattern, that for adding 3 digits numbers with 3 digits numbers we obtain a 4 digit number, so 3 bytes + 3 bytes = 4 bytes. This means that we always need to have one more compartment in the final tube, than the number of bytes which we've added. We decided to start with a simpler example, just to add 2 bytes with 2 bytes and create an initial sketch from which we could've developed a more complex system for adding numbers with more bytes.

After doing the coding process in the binary system, we've identified just three numbers which are coded in the binary system that have just 2 digits. The numbers are 1, 2 and 3 which coded in base 2 would be 1- 1; 2-10 and 3-11. Playing with these three numbers, we've identified a pattern. When adding one byte with one byte we need to obtain 10, so we have the result in two of the final compartments; when adding 1 byte with zero byte we need to obtain 1, the result in just one final compartment; when adding 0 byte with 0 byte we need to obtain 0 in just one compartment. The results in the final compartment means the number of balls which fall in it. For instance when adding 1+1= 10 so the result extends to two final compartment so in the first one we need to have 0 balls and in the second compartment 1 ball; for adding 1+0=1 we have the result only in one compartment and it need to have 1 ball inside; and when adding 0+0= 0 we have no balls in the final compartment.

Further we needed to place the "L gates" in such manner that the principle is followed and no mistakes are made. With the use of the so called "Logic Gates"* we are able to check the principle in the physical system as well. Based on the examples explained above, when we add 1 byte and 1 byte we need to obtain the final result 10 so we need to have a certain set of "L gates" which execute this task. So to prove that we first started with a simple system in which we need to incorporate the two "L gates", the regular gate and the 3 compartments in which the balls will gather/ collect. Different from the last system for the decimal base, we will group the "L gates" in other ways. Because we need to execute the test for 1 byte and 1 byte we will place the gates at the entrance of system, the groping process is the same and the "L gates" will be placed at the entrance of each tube corresponding to the first digits of the coded numbers. The “L gates” will then control a regular gate which is placed at the top of the first compartment of the system. The initial position are represented in the picture bellow. How the system works is pretty simple. The “L gates” will only move if there is just one ball introduced into the entering tube and the regular gate which is interconnected with the “L gates” only opens the first compartment when the single ball introduced pushes any one of the two” L gates” (it depends on which number has 1 byte as the first digit and which has 0 as the first digit). The regular gate won’t open when we have 1 byte and 1 byte because the “L gates'' will be simultaneously pushed and the rope won’t be able to move in any direction, it will only allow the 2 balls to fall through the system and pass the first compartment and then exit the system through a tube which isn’t continued nowhere.

Moving to the second set of tubes, which contains in adding the second digits of the coded numbers, we don’t have any gates, we just have at the rest of the main tube, two compartments which don’t have any normal gates and that is it. If we have balls falling through the second set of tubes then the balls will fill the two compartments.

When we’ve researched our theoretical approach for the binary system we’ve set out to extend our research in as many ways possible, we’ve first approached the basic addition with 2 bytes then we’ve realised a system which was able to add 3 bytes with 3 bytes in a similar way as for the 2 bytes sketch and lastly we’ve managed to create a system which combined the two, meaning, addition with 2 bytes and 3 bytes in the same system. As we’ve realised , adding the same number of bytes in both coded numbers was creating a repetitive system with similar rules and principles.

Let us move on to the system which makes possible the addition of 3 bytes and 3 bytes, the principle in which we’ve realised the addition for base 10 is followed in the binary system as well. We are going to use the already existing system for the 2 bytes addition and just add to it another set of tubes, a total of 3 now, and another final compartment. After doing this we realised that this won’t be enough to make the addition possible so we needed to add another few “L gates” using the same principle for logic gates used above. After we’ve kept the identical system for the addition for numbers with 2 bytes and added the tube after the initial system, on the last branch of the system, the third set of tubes, we’ve added the same logical gate the “AND” which we’ll follow the same test as in the first binary system. It will contain of two “L gates” interconnected with a rope which controls the added final compartment from the final tube, these regular gate will open only if we introduce in the third tube 1 ball or no ball, for 2 balls so 1 byte and 1 byte the regular gate will remain closed and both balls will exit the system without changing anything in it.

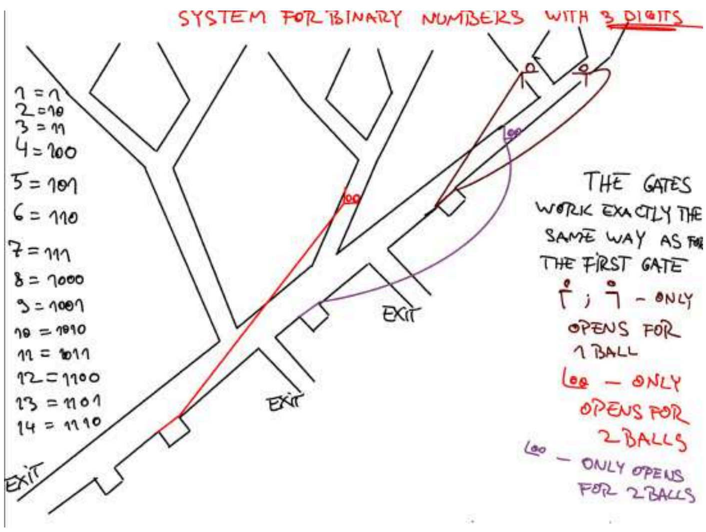


Figure 11: “Three-digit binary numbers”

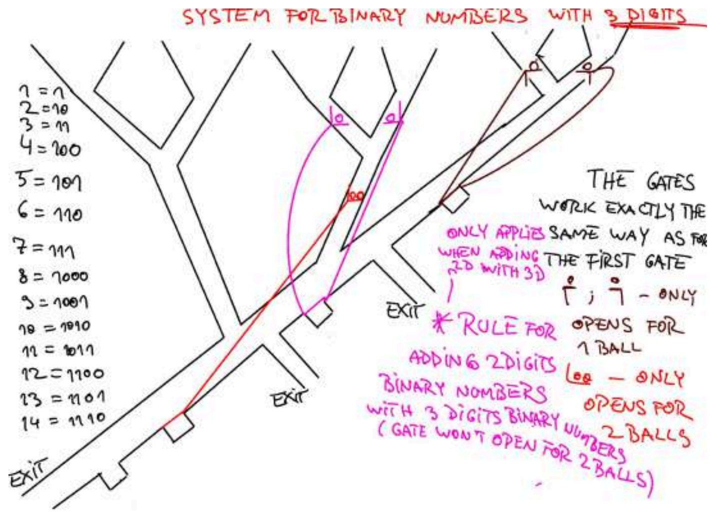


Figure 12: "Rules for the system"

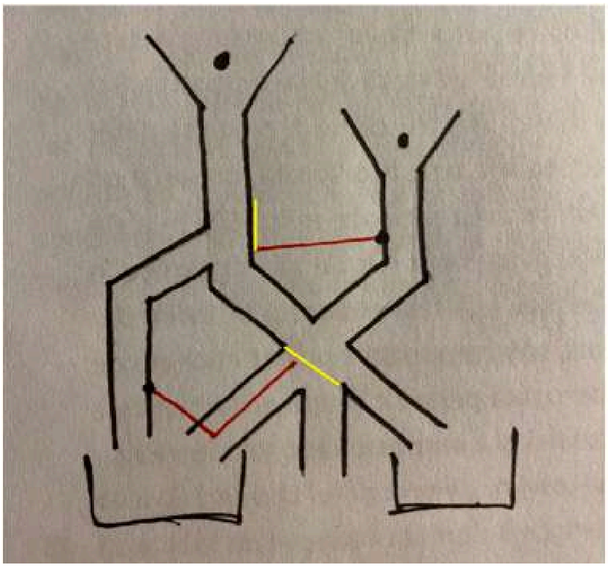
For adding numbers with more and more bytes we'll keep repeating this process, adding one more tube and a final compartment to the already existing system and adding the logic gates "AND" to execute the test for adding 1 byte and 1 byte. This is something which repeats itself for adding even numbers with 10 bytes and 10 bytes.

We've decided to research a more interesting way of combining the addition in the binary system. To do that we've decided to work on the addition of numbers with different quantities of bytes, for example adding numbers with 3 bytes and numbers with 2 bytes and achieving in the end a code number with 4 bytes. It's clear that the number of tubes in total need to be equal to the highest number of bytes (in our case 3), and the number of final compartments needs to be the highest. Starting from here we start to build a similar system as the one we've made for adding numbers with 3 bytes and 3 bytes. The same number of tubes and of final compartments. The principle we follow is just the same when adding 1 byte and 1 byte we need to obtain 10, so it is essential to introduce this component in our new system. On the third tube *(from the left) we'll introduce the two "L gates", the logical " AND" gate, which will control the regular gate which closes the first final compartment. The gate follows the same rule, it will open only if we have just one ball introduced down the tube. Moving on to the second tube, different from the 3 bytes system.

*Logic gates are the so-called process in which the computer executes different tests to check if he has 0 or 1 byte so that he will execute certain commands.

2.2.2 Second base 2 solution

This research follows the idea of coding the two numbers in base two and using them (in the form of beads) in the system. The solutions work for numbers up to two digits in base 2 (meaning 1, 2 and 3). So let the numbers be in the form ab and cd. We use the basic mathematical method of adding two numbers: we firstly do the sum of b and d, call it f, then the sum of a and c, call it e. The problem occurs when f>=10 (in base 2), when we need to add the 1 to a+c and follow the same rule.



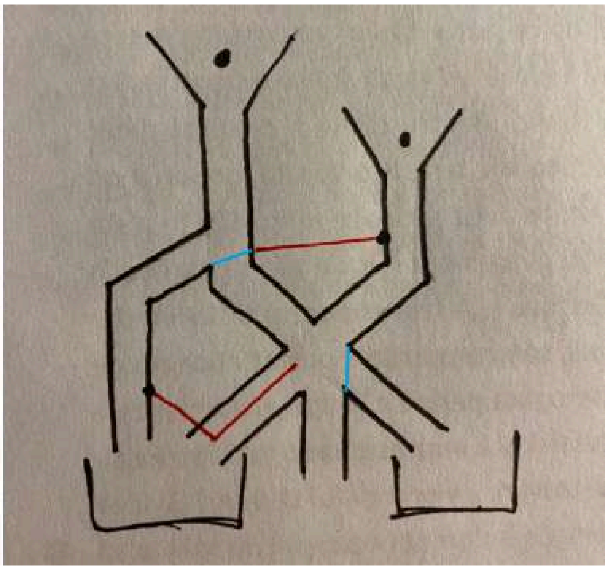


Figure 13.1 “Base 2 gates’ initial position *Figure 13.2 “Base 2 gates final” position”*

<u>Sum of two digits:</u> The addition is for a+c and b+d (from the numbers first named); The gates are activated by the falling of a ball and hitting one of the two black dots, the string which activates the gates is marked with red. The initial position of the gates is colored with yellow and the final one with blue (fig. 13.1 and fig. 13.2).

Example: When the sum of he two numbers is 10, a=0, c=0, b=1, d=1; The b ball follows the pink line and d the green one (fig. 14 right):

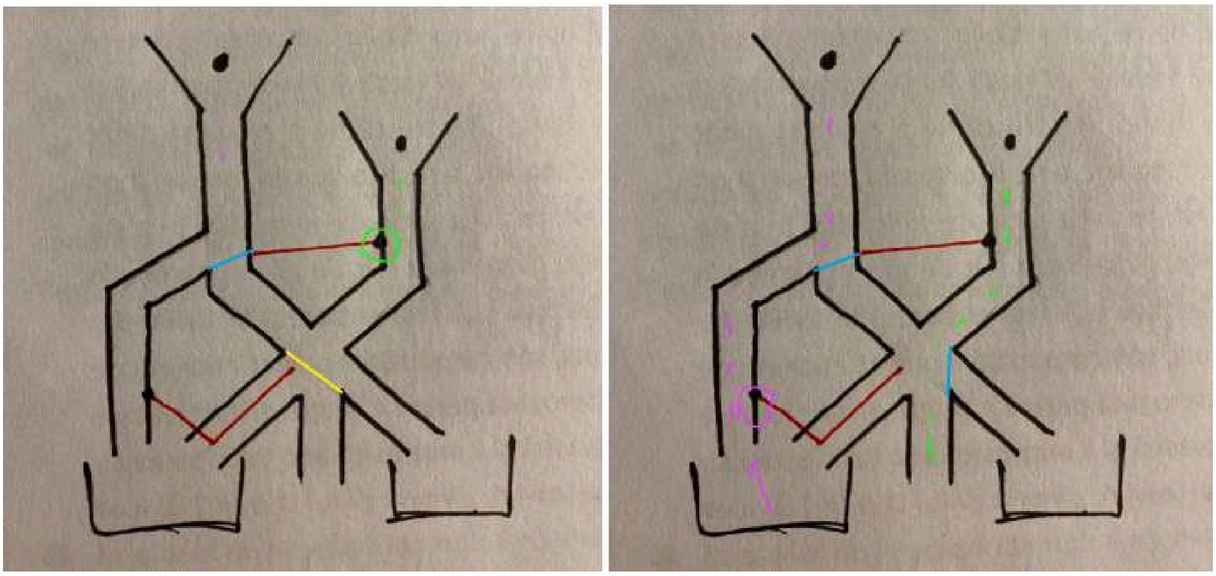


Figure 14 “Pink and green lines”

<u>Explanation</u> (fig. 14): 1. d falls and activates a gate which changes b’s way.
2. b also activates a gate which changes d’s way and ends up in the first box.
3. d falls out of the system.

Following the same path, I tried breaking down the sum of the two two digits numbers into the sum of two sums, meaning the sum of e and f. It does, however, not follow the same rules, the logic gates for the “big sum” do not respect the initial placement. The problem occurs when the ball that falls out of the system for b+d must be used after, in the example with the result 110 and so, must enter again. a and c follow the pink line, when b and d follow the green one.

Example for a=b=c=d=1 in three steps (fig. 15.1, 15.2 and 15.3):

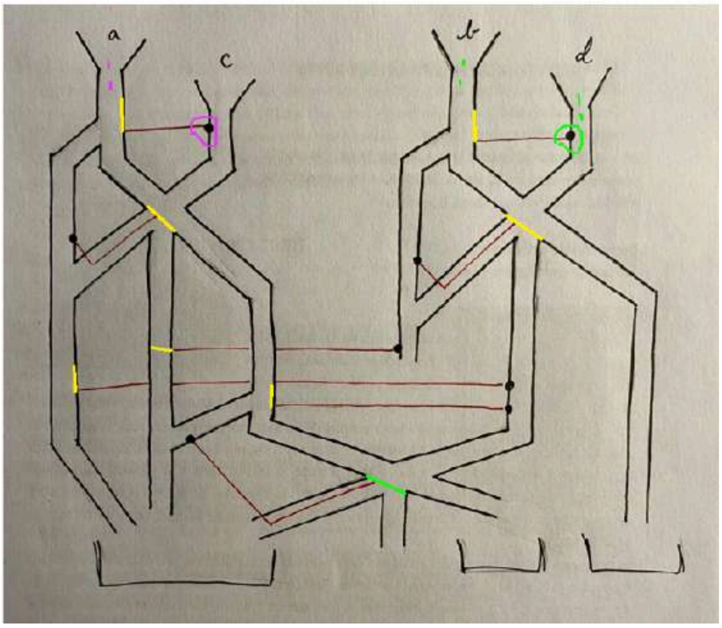


Figure 15.1: "First step"

Explanation (fig. 15.1): 1. a and d hit the buttons that change a's and b's path.

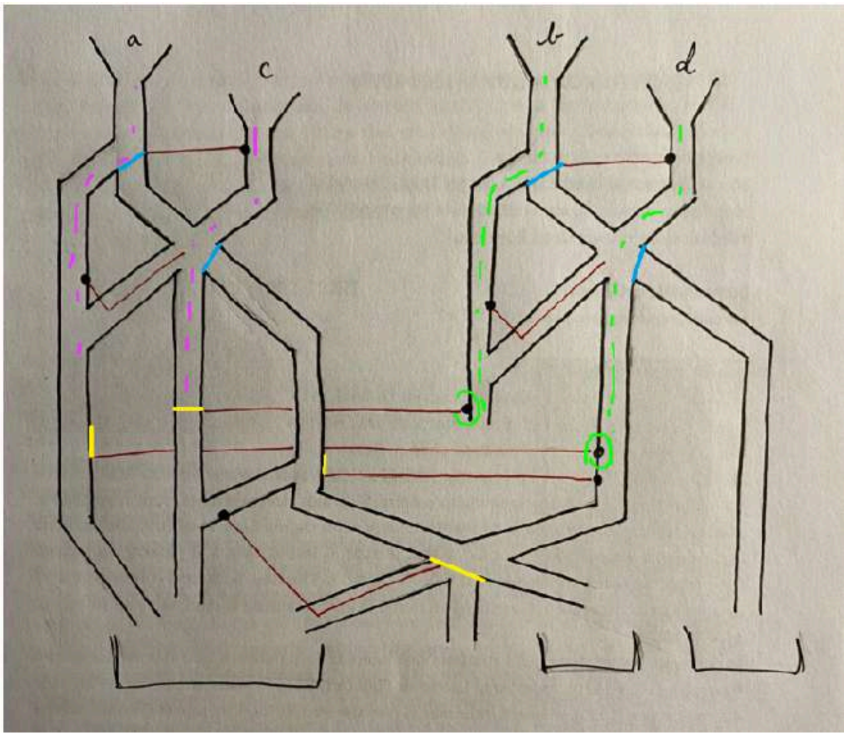


Figure 15.2: "Second step"

Explanation (fig. 15.2): 1. b and d hit the buttons marked with green, after which b falls out of the system.

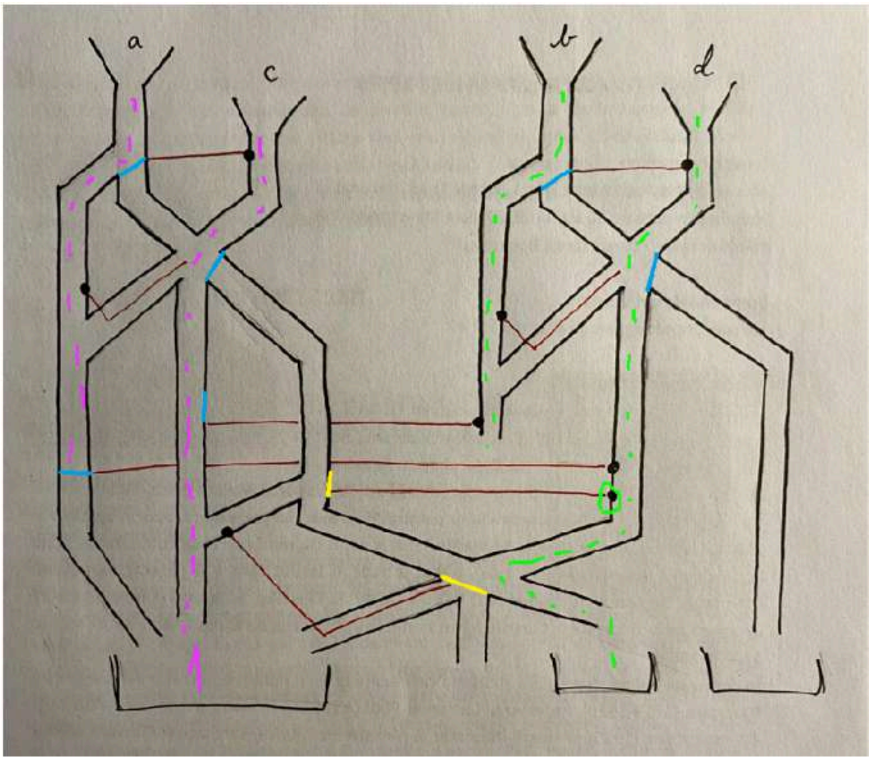


Figure 15.3: "Third step"

Explanation (fig. 15.3): Because of b, c is now falling free and ends up in the first box, whereas a is blocked by the gate activated by d. d is hitting another button which does not affect this case and falls in the second box. The final result is, therefore, 110.

This system changes again for numbers with digits greater or equal with 3. I assume the tube which removes the bead (between the first and second box) must be included again, as did the one above in this case, so that the bead can be used again. The system for that chase is not similar to this one.

The purpose and use of this project

The main purpose of this project is to help Visual-Spatial Learners understand mathematics in a different way, based on seeing the numbers, not imagining them. Being a visual learner means thinking in images instead of words, meaning that these people learn better off of diagrams, tables, maps and so on. Visual learners learn best by utilizing graphs, tables, charts, maps, colors and diagrams. They also tend to learn holistically, instead of sequentially, or in parts.

Conclusion

In the last few months each team has found and developed various solutions to the given problem.
Thus, we currently have a phisical model of the system and know several ways to solve the problem in two numeration bases.

Nevertheless, the research can be continued and expanded to more numeration bases or, pheraps, with the purpose of finding a system that can solve the problem no matter the base that the two numbers are coded in.